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COURSE NOTES
"SHEAR STRENGTH BEHAVIOR OF COHESIVE SOILS"
1961 SUMMER PROGRAM
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

COMPILED
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TECHNICAL REPORT

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I. INTRODUCTION

During the past ten years there has been a large amount of research work done to gain an understanding of the shear strength behavior of cohesive soils. We have read with interest some of the published papers, been confused by others, and have not had time to read most of them.

These papers contributed to the development of the understanding of one of the most important concepts in soil mechanics - THE EFFECTIVE STRESS PRINCIPLE. This principle is not new as it was first conceived by Terzhagi some 35 years ago. It is the hypothesis that shear strength is best described by the effective stress on the failure plane and a knowledge of the pore water pressure in the soil mass. However, it received a tremendous impetus about ten years ago when the English (Skempton, Bishop, and Henkel) and the Norwegians (Bjerrum) applied this principle as a key to explain the strength and settlement characteristics of cohesive soils. At the present state of development, this principle has been expressed in theoretical form and has been demonstrated by laboratory tests. Now the research people and engineers are turning their attention to the field to try out these concepts in practice which will be the ultimate test.

The effective stress principle cannot be dismissed as the "newest fad" or being impractical. It allows the engineer

to improve his knowledge of the behavior of soils under all conditions in which they exist in nature. This fact alone should put engineering judgment on a sounder basis. It is strongly urged that all soils engineers study these notes which attempt to present the basic ideas for understanding the effective stress principle.

Let nobody get the misconception at this time that the effective stress principle is going to revolutionize our testing and design methods. However, I believe that if you study and absorb the information in these notes you will have a better understanding of the soil you work with everyday.

II. PRESENTATION OF COURSE CONTENT

The following outline is a condensed presentation of the material covered in the course. Printed notes which give the material in much more detail were handed out and they are available for your use. Much of the course content is also presented in the ASCE publication "Research Conference on Shear Strength of Cohesive Soils" - Boulder, Colorado, 1960.

III. THE EFFECTIVE STRESS PRINCIPLE

A. General Definition

In settlement and stability problems it is essential to apportion the total stress carried by the soil mass into the portions carried by the various phases i.e. soil, water and air. This apportionment of total stress leads to the most important principle in all of soil mechanics, namely, the EFFECTIVE STRESS PRINCIPLE. This principle

consists of two parts: 1) an equation (or equations) apportioning the stress carried by the different phases, and 2) expressions relating soil behavior with the various phase stresses.

B. Measuring the Stresses Involved.

1. Finding stresses at a point in the ground.

   a. Static Case - This is no problem as it is simply determining the stresses from the weight of soil and water above the point at depth.

      σ (total pressure or stress) $= z\,\gamma_t$

      u (pore pressure) $= z\,\gamma_w$

      $\bar{\sigma}$ (effective stress) $= \sigma - u$

   b. Non- Static Case - The pore pressures will change due to 1) steady seepage forces or 2) applied stresses to the soil mass.

2. Determination of stresses

   a. The total pressure (σ) can be determined from the measured or calculated pressures acting on a point either in the ground or in the laboratory specimen.

   b. The pore pressure (u) can be determined by calculation in some cases or by pore pressure measuring equipment in all cases in the field or laboratory.

   c. The effective stress ($\bar{\sigma}$) is determined indirectly from $\sigma - u$. (1)

(1) For further information see Course Notes - Part J entitled "The Elements of Fluid Flow in Soils".

consists of two parts: 1) an equation for specifically apportioning the stress carried by the different phases and 2) expressions relating soil behavior with the various phase stresses.

5. Reviewing the stresses involved.

1. Finding stresses at a point in the ground.

a. Static Case - This is no problem as it is simply determining the stresses from the weight of soil and water above the point at depth.

σ (total pressure or stress) = $\Sigma \gamma$

u (pore pressure) = $\Sigma \gamma_w$

$\bar{\sigma}$ (effective stress) = $\sigma - u$

b. Non-Static Case - The pore pressures will change due to 1) steady seepage forces or 2) applied stresses to the soil mass.

2. Determination of stresses

a. The total pressure (σ) can be determined from the measured or calculated pressures acting on a point either in the ground or in the laboratory specimen.

b. The pore pressure (u) can be determined by measuring it when possible by pore pressure measuring equipment in all cases in the field or laboratory.

c. The effective stress ($\bar{\sigma}$) is determined indirectly from $\sigma - u$.[1]

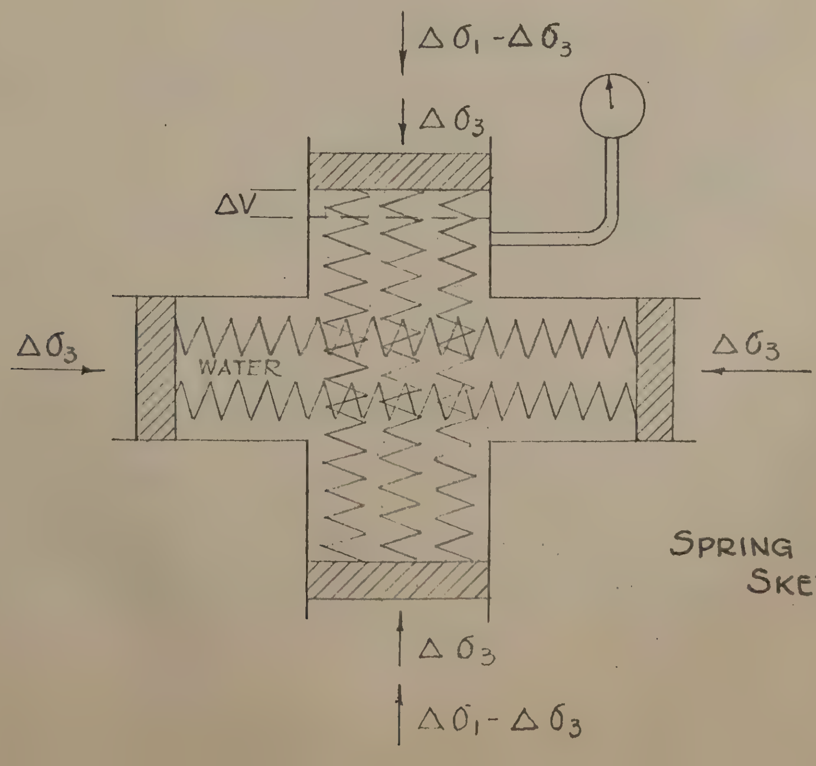
(1) For further information see Journal Notes - Part I entitled "The Mechanics of Fluid Flow in Soils".

IV. THE PORE PRESSURE PARAMETER CONCEPT

A. Spring Analogy

The stress conditions in a soil are best understood by visualizing it as a compressible skeleton of solid particles enclosing voids which, in saturated soil are filled with water, or, in partly saturated soil filled with both air and water. Shear stresses can, of course, be carried only by the skeleton of solid particles. On the other hand, the normal stress on any plane is, in general, the sum of the stress carried by the solid particles and the pressure in the fluid in the void space.

Let us observe what happens when stresses are applied to a soil system. The following sketch is an analogy to a soil-water system. The container is filled with water and has springs in both directions to simulate the soil structure. A gage measures the water pressure.



SPRING ANALOGY
SKETCH A

1. A pressure $\Delta\sigma_3$ is applied to all sides. The pressure gage records a change in water pressure of $\Delta\sigma_3$. Therefore $\Delta U = \Delta\sigma_3$. Note that the springs do not compress because pressures are equal on all sides.
2. Next a pressure is added to the top piston which we will call $\Delta\sigma_1 - \Delta\sigma_3$. Now since the external pressures are not equal on all sides of the container this applied pressure is carried partly by the spring system representing the soil skeleton and partly by the water. Therefore, the pressure gage will read only a fraction of the additional applied pressure. This fraction is defined as pore pressure parameter A. The total pressure recorded by the gage is expressed as follows:

$$\Delta U = \Delta\sigma_3 + A\,(\Delta\sigma_1 - \Delta\sigma_3)$$

$$A = \frac{\Delta U - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3}$$

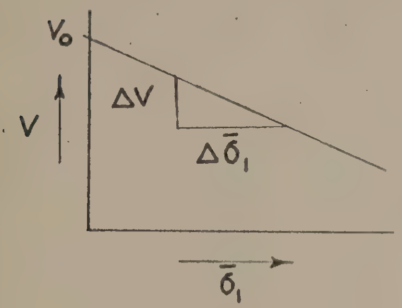
This is the simplest demonstration possible of the pore pressure parameter concept. This is the parameter for three dimensional uniform loading followed by one dimensional loading such as occurs in the triaxial test. FOR THE CONSOLIDATED - UNDRAINED TEST, THE INITIAL $\Delta\sigma_3 = 0$. THEN $A = \frac{\Delta U}{\Delta\sigma_1 - \Delta\sigma_3}$ OR $\frac{\Delta U}{\text{DEVIATOR STRESS}}$. THIS WILL BE THE MOST COMMON APPLICATION FOR PRACTICAL PROBLEMS.

B. Determining Pore Pressure Parameter For One-Dimensional Consolidation.

At this point one can visualize in the spring analogy that the amount of deflection of the spring under load and consequent increase in pore pressure will depend on the spring constant. Substituting the soil skeleton for the spring we can see that the amount of compression would

depend on the compressibility of the soil skeleton.

If we place a sample of dry soil in a consolidometer and run a consolidation test with free drainage we can determine the compressibility of the soil skeleton as follows:



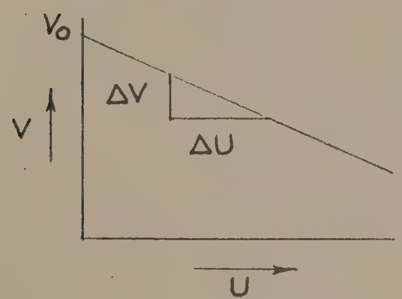
ONE DIMENSIONAL COMPRESSION OF SOIL SKELETON

The slope of the Compression Curve is: $-\dfrac{\Delta V}{\Delta\bar{\sigma}_1}$

Expressed in Unit Strain, it is:

$$C_{c_1} = -\frac{\Delta V}{V_o}\cdot\frac{1}{\Delta\bar{\sigma}_1}$$

We can also theoretically find the compression characteristics of water. The consolidometer is filled with water and the loading conducted with no drainage allowed to give the following results:



ONE DIMENSIONAL COMPRESSION OF WATER

The slope of the Compression Curve is: $-\dfrac{\Delta V}{\Delta U}$

Expressed in Unit Strain, it is:

$$C_w = -\frac{\Delta V}{V_o}\cdot\frac{1}{\Delta U}$$

Now let us take the same soil sample, fill the voids with water, place it in the consolidometer and run a loading test with <u>no</u> drainage allowed. Therefore, any

V_0 = median size of sample

= [illegible]

= % of sample which is void

change in the volume of the soil skeleton must be equal to the change in volume of the pore fluid. $\Delta V_{SK} = \Delta V_P$

For change in volume of the soil skeleton we have

$$C_{c_1} = -\frac{\Delta V}{V_0} \cdot \frac{1}{\Delta \bar{\sigma}_1}$$

$$\Delta V = \Delta V_{SK} = -V_0 C_{c_1} \cdot \Delta \bar{\sigma}_1 \qquad \text{(Equation 1)}$$

For change in volume of pore fluid we have

$$C_w = -\frac{\Delta V}{V_0} \cdot \frac{1}{\Delta U}$$

$$\Delta V = -V_0 C_w \Delta U$$

Since the volume of water in the soil is now equal to the volume of voids we have

porosity $\frac{V_v}{V}$

$$\Delta V_P = -n V_0 C_w \Delta U \qquad \text{(Equation 2)}$$

tells % of total sample that is pore H

Now we can equate the change in volumes from equations 1 and 2.

$$-V_0 C_{c_1} \overline{\Delta\sigma_1} = -n V_0 C_w \Delta U$$

$$\overline{\Delta\sigma_1} = \frac{n C_w \Delta U \cancel{V_0}}{C_{c_1} \cancel{V_0}} \qquad \text{(Equation 3)}$$

By definition

$$\bar{\sigma} = \sigma - U$$

$$\Delta \bar{\sigma} = \Delta \sigma_1 - \Delta U$$

Then substituting equation 3 for $\Delta\bar{\sigma}$

1. $\frac{n C_w \Delta U}{C_{c_1}} = \Delta \sigma_1 - \Delta U$

2. $n C_w \Delta U + \Delta U C_{c_1} = \Delta \sigma_1 C_c$

3. $\Delta U (C_{c_1} + n C_w) = \Delta \sigma_1 C_c$

4. $\frac{\Delta U}{\Delta \sigma_1} = \frac{C_{c_1}}{C_{c_1} + n C_w} = \frac{1}{1 + \frac{n C_w}{C_{c_1}}}$

5. Pore Pressure Parameter $= \frac{\Delta U}{\Delta \sigma_1}$

The parameter for one dimensional consolidation is called C. Therefore

$$C = \frac{\Delta U}{\Delta \sigma_1} = \frac{1}{1 + n\dfrac{C_w}{C_{c_1}}}$$

C. Comments on Pore Pressure Parameter

On Figure 1, (pg. 10) the pore pressure parameters are presented for various conditions of loading. A different parameter exists for each condition as the strains in the sample will vary.

If the soil behaves as an elastic isotropic material and the fluid in the pore space shows a linear relation between volume change and stress, then the pore pressure may be expressed in terms of Poisson's Ratio (E) and Young's Modulus (μ) for the soil.

Note that Lambe has defined the parameters in the following manner.

Triaxial Loading----------------------Parameter A

Three Dimensional Uniform Loading-----Parameter B

One Dimensional Compression-----------Parameter C

One Dimensional Loading---------------Parameter D

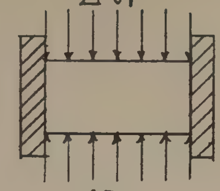
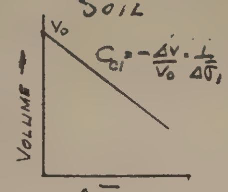
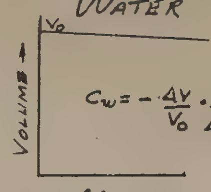
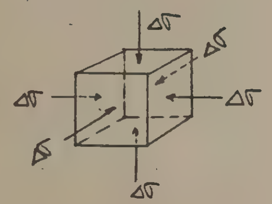
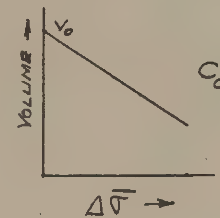
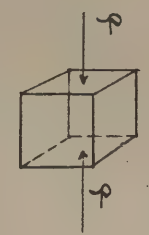
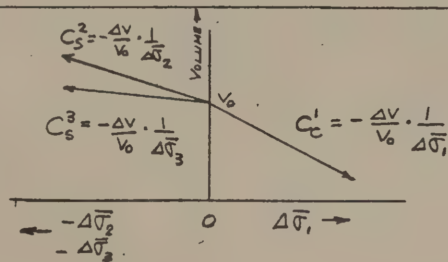
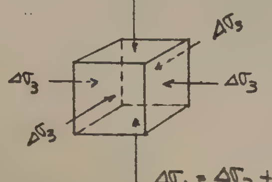
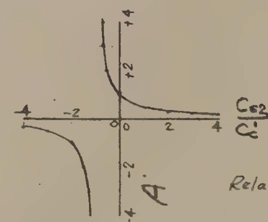
Some confusion exists on nomenclature as Bishop and Henkel use B to denote the case of triaxial loading with both water and air in the pores. It is suggested that we use Lambe's nomenclature.

Do not picture the pore pressure parameter as being a constant value for each soil and each loading condition. It will vary during a shear test depending on the rate of

strain and magnitude of strain. The parameter values will also be influenced by sample disturbance, preconsolidation history, rotation of planes of principal stress, and the soil structure.

Remember that a pore pressure parameter is a dimensionless number which indicates the fraction of total stress increment which shows up as excess pore pressure for the condition of no drainage - i.e. constant mass. It is thus a number which permits one to separate the total stress increment into the components carried by the different phases of the soil system. We will return to the pore pressure parameter later.

Pore Pressure Parameters

One-Dimensional Compression	$\Delta\sigma_1$  $\Delta\sigma_1$	Soil  $C_{c1} = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta\sigma_1}$; Water  $C_w = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta u}$ $C_c = \frac{3(1-2\mu)}{E}$	$C = \frac{\Delta u}{\Delta\sigma_1} = \frac{1}{1+n\frac{C_w}{C_{c1}}}$ Values may also be determined from Consolidation test
Three Dimensional Uniform Loading		 $C_{c3} = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta\sigma}$	$B = \frac{\Delta u}{\Delta\sigma} = \frac{1}{1+n\frac{C_w}{C_{c3}}}$
One-Dimensional Loading		 $C_s^2 = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta\sigma_2}$; $C_s^3 = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta\sigma_3}$; $C_c' = -\frac{\Delta V}{V_0}\cdot\frac{1}{\Delta\sigma_1}$	$D = \frac{\Delta u}{p} = \frac{1}{1+n\frac{C_w}{C_c'}+\frac{C_s^2}{C_c'}+\frac{C_s^3}{C_c'}}$ $D = \frac{1}{1+n\frac{C_w}{C_c'}+\frac{C_{s2}}{C_c'}}$, where $C_{s2} = C_s^2 + C_s^3$
Three-Dimensional Uniform Loading followed by One-Dimensional Loading	$\Delta\sigma_1 = \Delta\sigma_3 + (\Delta\sigma_1 - \Delta\sigma_3)$ validity of superposition questioned  $\Delta\sigma_1 = \Delta\sigma_3 + (\Delta\sigma_1 - \Delta\sigma_3)$ Duplicates Triaxial Test	$\Delta u = B\,\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3)$ $\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$ if $C_w \approx 0$ $\Delta u = \Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)$ if $C_c' = C_s^2 = C_s^3$, i.e. soil elastic	$A = \frac{\Delta u - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3} = \frac{1}{1+\frac{C_{s2}}{C_c'}}$  Relation of $\frac{C_{s2}}{C_c'}$ to A

Bjerrum uses $E = \frac{\Delta U}{Gc}$ to check work

FIG. 1

V. BASIC PRINCIPLES OF STRENGTH BEHAVIOR OF SATURATED SOILS

A. Basic Three-Way Relationship

It has been recognized for sometime that there is a definite relationship between soil strength; water content or void ratio, and effective stress. Since determination of effective stresses depends on determining the pore pressure, this portion of the relationship has not been fully utilized until recently. It was only fifteen years ago that good reliable techniques were developed for determining pore pressures.

The triangular chart below depicts the inter-relation of these properties of soil.

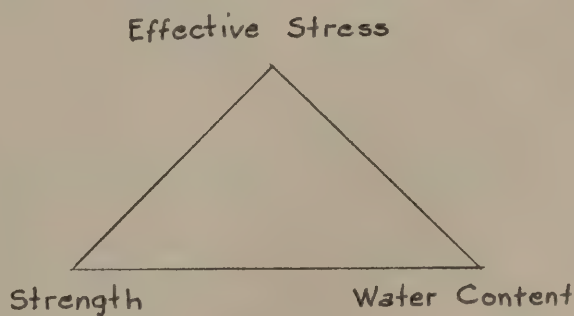


Figure No. 2 (pg.12) is a copy of chart in "Fundamental of Soil Mechanics" by Taylor. This shows the basic 3-way relation and also the relation between e-log P consolidation tests and e-log S_s plots of shear strengths of similar soils. One limitation to this relationship is that it is limited to normally loaded clays.

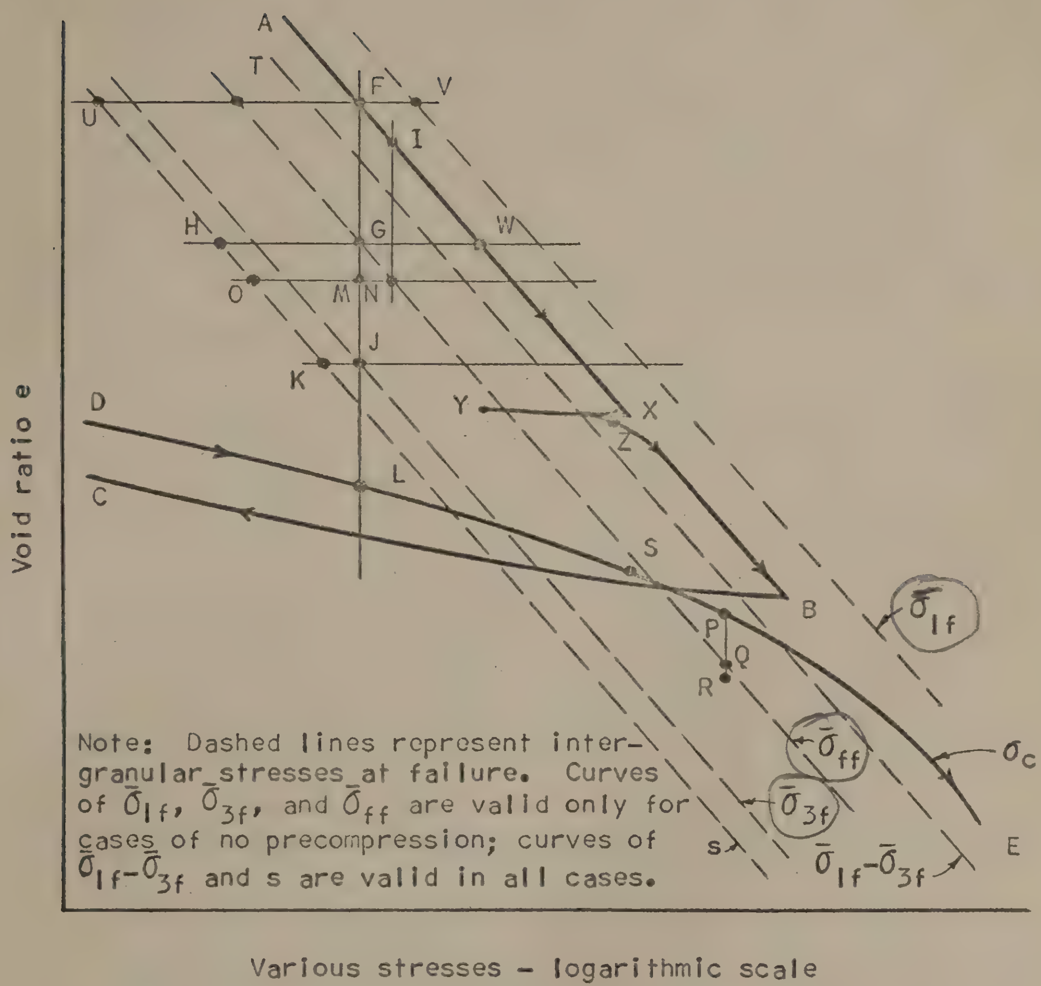
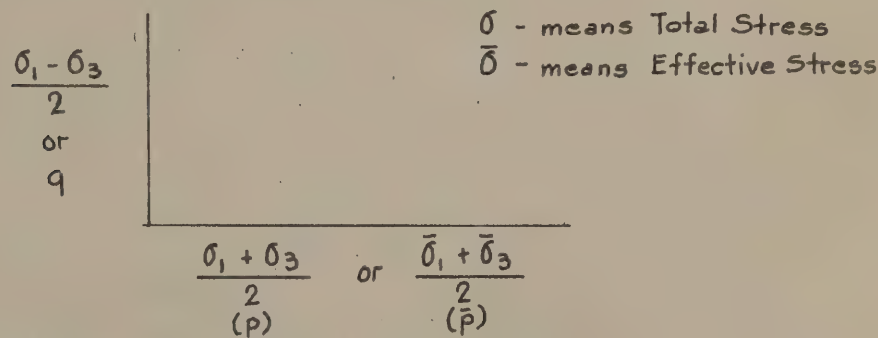


Fig. 2 Plot of void ratio versus pressures and strengths

B. Presentation of Test Data

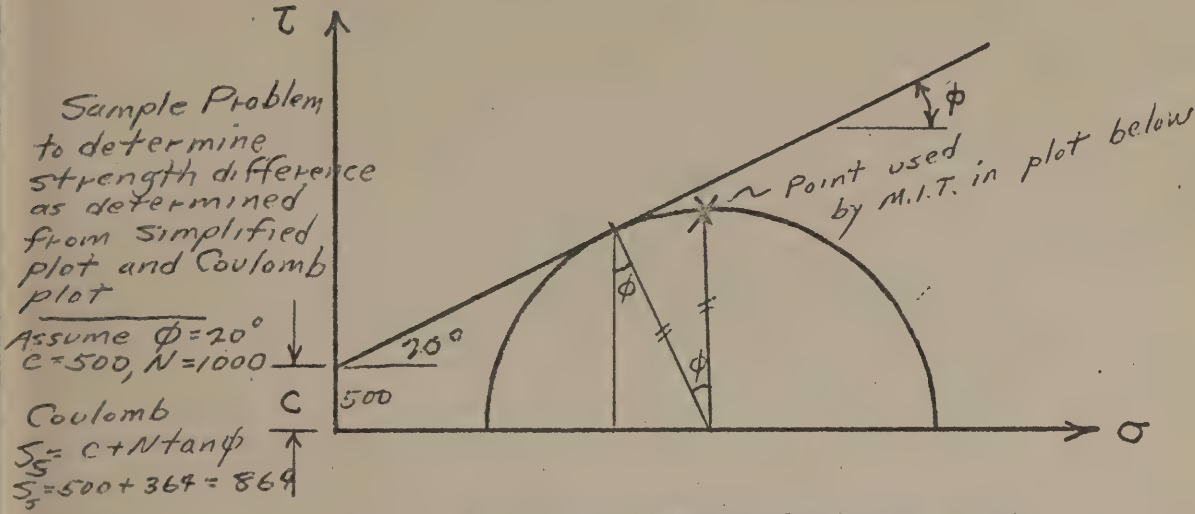
1. q vs. p Plot

All of the testing obtains data to relate the effective stress on the failure plane to the shearing strength. It would be very difficult to present this data on the usual Mohr circle plot. All the test data is presented in the following manner. The shear strength from a single test would be plotted as a point.

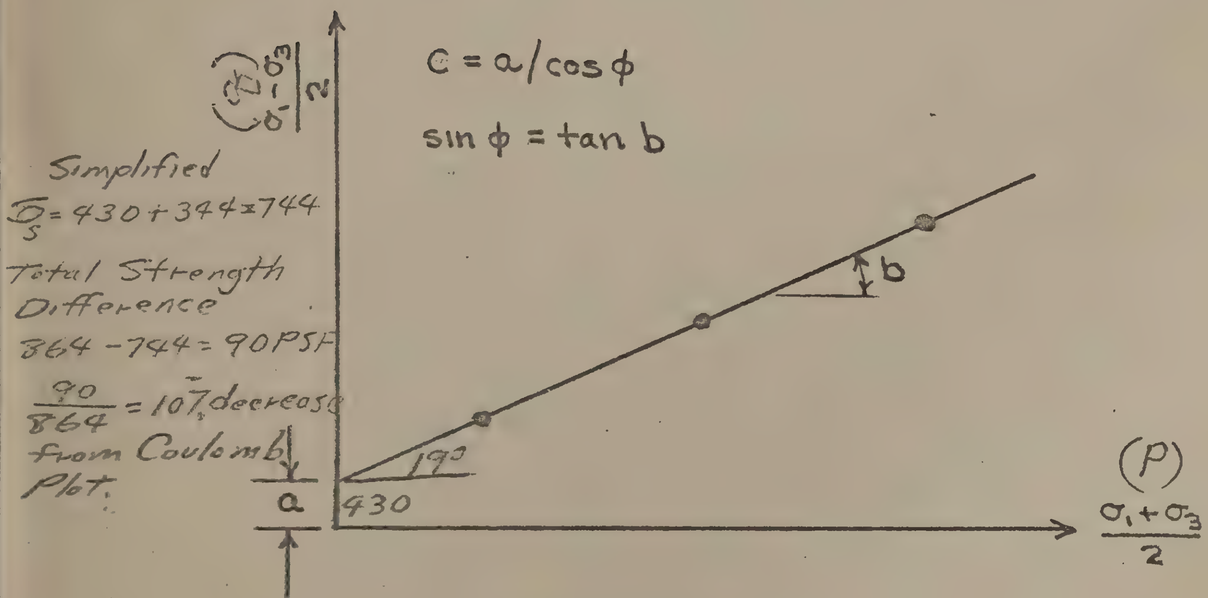


2. Comparison of Strength Results from Mohr Plot Compared to q-p Plot.

A study of Figure No. 3 (pg. 14) shows the relation between the strengths determined from the Mohr Circle plot and q-p plot. The sample problem worked out shows that if a shear strength is obtained at an overburden pressure of 1000 PSF then the shear strength using the simplified plot will be 10% less than that obtained from the Mohr Circle plot. For a soil with no friction angle the q-p plot would have a shearing strength about 10% higher than the Mohr Circle Plot. This raises the question of which one is correct. The following reasoning was presented:



Ordinary Mohr-Coulomb plot



Simplified plot

FIGURE 3

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a. The failure plane in a triaxial sample would not duplicate the failure plane and stresses on the plane of failure in the ground. This is not unreasonable when you consider how the principal stresses have to rotate across the arc of a slip circle. Observed failure plane slopes in field do not agree with sample failure plane in laboratory.

b. The internal mechanism of shearing resistance in soil bears little resemblance to the ϕ and C strength concept. This will be elaborated on later.

c. The many ignorance factors that have to be evaluated in stability analysis assumptions outweigh the importance of refinements on the friction angle and c intercept.

3. Vector Curves or Effective Stress Paths

One of the most important parts of the test interpretation is to plot the effective stress on the plane of maximum shearing stress vs. $\frac{\sigma_1 - \sigma_3}{2}$ at all stages of a test from beginning to failure. These points connected form a vector that traces the development of effective stresses on the failure plane. For further explanation this plot will be shown by Mohr Circle construction later on.

4. Strength Test Nomenclature

The soil mechanics world has finally agreed on a standard abbreviation system for strength test identi-

fication as follows:

CU	-	Consolidated - Undrained
CD	-	Consolidated - Drained
UU	-	Unconsolidated - Undrained
U	-	Unconfined Compression

5. Review

Know the plots and nomenclature in this section before going on or you may get lost.

VI. A REVIEW OF THE STRENGTH BEHAVIOR OF SATURATED SOILS

A. Summary

The purpose of this section is to demonstrate by a review of shear test results that the shear strength of a "cohesionless" soil such as sand and the shear strength of a "cohesive" material such as clay will be determined by the drainage conditions within the soil mass. In other words the strength of a soil is determined by the effective stress which varies with the pore water pressure within the soil mass.

This explains why a clay slope after many years will assume an angle of repose similar to that expected from a cohesionless soil. It also explains why sand and silt deposits will suddenly fail in a flow slide in a manner that would be expected from a sensitive clay.

How does this tie in with our present concept of thinking in terms of friction angle and cohesion? For most soils problems involving granular soil the friction angle concept is valid because we have a consolidated

drained material and the available strength increases with load. For cohesive soils we have consolidated - undrained conditions and the available strength does not increase appreciably with load. Note the key words are drained and undrained. The permeability of a soil is an important factor in its strength behavior. Therefore, in order to explain the strength behavior of all soils under all conditions of drainage in nature we have to attack the problem from the effective stresses which vary with the pore pressure in the soil mass. This approach can be an important tool to gain an understanding of some of our more difficult soils problems involving stability and settlement.

B. Behavior of Saturated Sands (See Figure 4) (Pg. 25)

Two samples of saturated sand are consolidated at different pressures. Both samples are sheared without drainage and they both have the same strength. A third sample is prepared with air instead of water.

The results show that the shear strength is determined by the effective stresses, not the total stresses. The presence of air instead of water in the voids does not alter the relationship between shear strength and effective stress.

C. Behavior of Normally Consolidated Clay

1. Consolidated - Drained Tests

Three samples are prepared and consolidated by a pressure of $\bar{\sigma}_3$ as shown on Figure No. 5. Triaxial

compression tests are conducted allowing the samples to drain as load is applied. The stress-strain curves are shown as CD on Figure No. 6. For the three tests the maximum shearing strength $\frac{\sigma_1 - \sigma_3}{2}$ is plotted vs. the effective stress on failure plane ($\bar{p}_f$) as shown on Drawing No. 7. The three points are connected to form the q_f vs. p_f line. This envelope is similar but not exactly in the same location as the Mohr circle envelope. This relation was previously shown.

Next the effective stress path is plotted. The effective stress is determined for intermediate Mohr circles before failure. The effective stress path is made by connecting these points.

2. Consolidated - Undrained Tests

These samples are prepared and consolidated under a pressure of $\bar{\sigma}_{3_c}$. The load is applied with no drainage and the stress-strain curves are shown on Figure No. 6. Plots of q_f vs. $\bar{p}_f$ similar to those for the CD test are shown on Figure No. 8.

Note that the effective stress vector has an entirely different shape than for the CD plot. This is because $\bar{\sigma}_3$ at any stage during the test is equal to $\bar{\sigma}_{3_c}$ at consolidation minus ΔU developed. Since pore pressure or ΔU increases with load then $\bar{\sigma}_3$ decreases with load.

Note that a relation exists between moisture content and q_f, $\bar{p}_f$ & $\bar{p}_o$. This ties in the strength, effective

stress, and moisture content triangular relationship.

3. Theory Connecting Drained and Undrained Tests

The key to behavior of saturated soils lies in the difference between CD and CU tests and in the link provided by effective stress. Following is developed a useful relationship.

If two specimens of a given clay are consolidated to the same stress $\bar{p}_o$ and one is sheared with drainage and the other without further drainage different values of q_f will result. Since the ratio $\frac{q_f}{\bar{p}_f}$ must be the same in the two specimens, there must be a relation between the pore pressure developed in the CU test and the strength measured in that test.

Using the pore pressure parameter $A = \frac{\Delta U}{\sigma_1 - \sigma_3}$ it is possible to derive an equation relating $\bar{\phi}$ and the $\frac{q_f}{\bar{p}_o \text{ or } \bar{p}_f}$ ratio. This is done in Figure 9. For future reference, the derivation makes provision for a cohesion intercept. Values of A_f for normally consolidated clays usually fall in the range from 0.5 to 1.0. The A_f value for the clays used in this testing was determined as 0.83. Figure 10 shows how the pore pressures during the test may be obtained by graphical constructions on the q vs. $\bar{p}$ plot. This plot also shows how the pore pressures develop during a test.

4. The $\phi = 0$ Concept (UU Test)

Three samples are consolidated at 16 PSI ($\bar{\sigma}_3$)

and sheared undrained at pressures of 10, 30 and 100 PSI (σ_3) . The result plotted on Figure No. 11 is a constant q_f value of 4.8 PSI for each sample. Note that pore pressures are negative. The strength and effective stress are the same for each sample. This shows undrained strength is independent of changes in the total stress P.

5. U-Test

This test is similar to the UU test with a negative pore pressure. Plot the effective stress path and ΔU for this case.

6. Typical Values of $\bar{\phi}$ for Normally Consolidated Clays

Figure 12 shows the summary of available data. ($\phi' = \bar{\phi}$) There is a trend toward decreasing ($\bar{\phi}$) with increasing plasticity. The ($\bar{\phi}$) for very plastic soils is uncertain and the effective stress concept today is not applicable to extremely plastic clays.

7. Typical q_f vs p_o Ratios for Normally Consolidated Clays

The undrained shear strength should increase in depth as a result of the increase in effective stress with depth. Figure 15 shows this increase in a summary of $\frac{q}{\bar{\sigma}_1}$ ~~of~~ ratios (referred to as c/p) as a function of plasticity index. Recent testing on sensitive marine clays show a decrease at lower plasticity values. This lower branch may be partially explained by the following section.

8. Sensitive Clays and Sands

Sensitive (or quick) clays and extremely loose saturated sands exhibit a very important form of deviation from the concept of a unique $w_f - q_f - \bar{p}_f$ relation. This deviation is so important that it must at least be introduced in this elementary presentation.

Figure 13 shows a typical stress-strain curve for an undrained test. As contrasted to the stress-strain curves we have examined previously, this one shows a pronounced peak. Following this peak, the undrained strength of the clay drops rapidly. In extreme cases, such a clay, when only slightly disturbed, assumes the consistency of a thick liquid. Such is the origin of the spectacular flow slides of Scandinavia and the Canada - U.S. border area. The same type of stress-strain curve is obtained from CU tests on extremely loose, fine sands (BJERRUM, KRINGSTAD and KUMMENEJE, 1961), and flow slides have also occurred in such sands.

Figure 13 and 14 shows the results of such a test on a q vs. $\bar{p}$ plot. The effective stress path reaches a peak before the maximum obliquity condition is reached. The extreme end point of the CU effective stress path gives the same $w_f - q_f - \bar{p}_f$ relation as a CD test, but the points of maximum stress difference in the two tests fall into different $w_f - q_f - \bar{p}_f$ relations.

These phenomena develop in clays and sands that have a meta-stable structure; i.e. the void ratio is larger than it would normally be for the given $\bar{p}_o$. Such meta-stable structure develops from very gradual deposition in quiet water. Such conditions make possible the growth of a "cement" between particles sufficient to hold the mineral skeleton in the loose state. Sensitive clays quite typically have a water content above the liquid limit. When the soil is sheared or otherwise disturbed, the "cementing" forces are overcome, the mineral skeleton starts to collapse, most of the initial effective stress is transferred to pore water pressure, and the shear resistance drops. A_f values as large as 2 and even 3 have been recorded. This behavior accounts for the low $q_f/\bar{p}_o$ ratios shown in Figures 14 and 15.

There is a growing body of evidence that this collapse can be seen in CD tests as well. As indicated in Figure 13, a yield point (sometimes followed by quite a plateau) appears in the CD tests for very loose sands. At this stage, since the sand is still quite loose, there is little or no geometrical interference and the friction angle is very low. With additional straining and a marked volume decrease, the particles nestle among one another and the angle of shearing resistance increases. The w_f - q_f - $\bar{p}_f$ relation from the yield point of a CD test does more

or less agree with the relation for the peak points of CU tests. Thus, by introducing the concept of a meta-stable structure with a yield point, a certain amount of unity can be restored to the shear strength picture.

The Norwegians have analyzed slides in sensitive clays and determined the effective $\emptyset$ at failure was 8^o. The minimum laboratory test friction angle was 15^o.

The field vane shear test is probably the only reliable method of measuring this increase in strength with depth.

D. Behavior of Over-Consolidated Clay

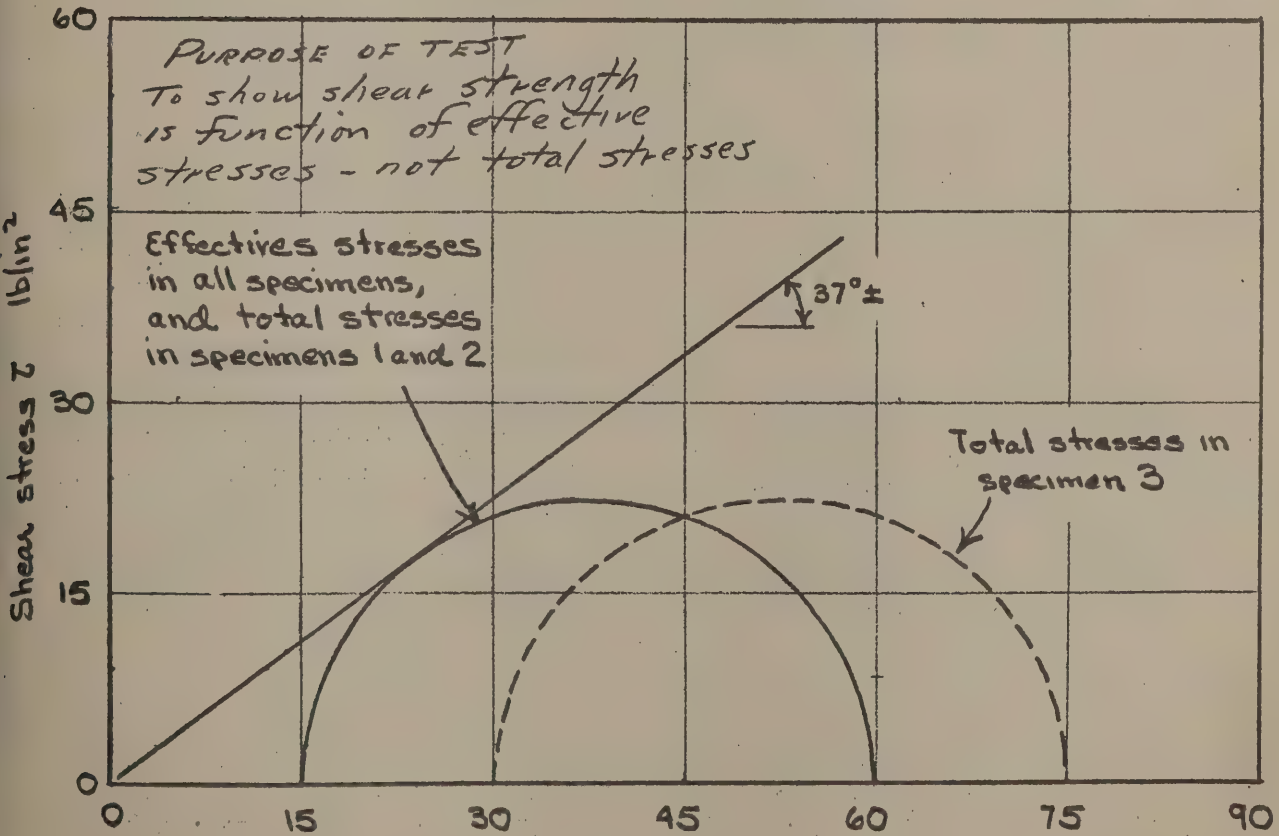
Over-consolidated clays tend to expand when sheared thus setting up negative pore pressures and an increase in strength. Laboratory experiments show that this increase in strength does not take effect as long as $\bar{p}_f$ is greater than 0.5 $\bar{P}m$ (max. past pressure). Figures 16, 17, 18 and 19 show the effects of over-consolidation on strength. When $\bar{p}_f$ is less than 0.5 $\bar{P}m$ then new w_f vs q_f relations apply. Figure 20 shows the variation of A_f with $\frac{\bar{P}m}{\bar{P}o}$ Using this curve it is possible to solve problems and determine the U_f at failure.

This increase in strength results in a $\bar{c}$ intercept for the q_f vs. p_f envelope. It is possible that the effects of preconsolidation may disappear entirely during an extremely slow loading process. There is some field

evidence that this happens in stiff fissured clays. On the other hand $\bar{c}$ may be permanent from cementation between particles rather than simply from the remembrance of consolidation. (1)

(1) The preceding material is abstracted from the Detailed Course Notes - Part A - entitled "Strength Behavior of Saturated Soils" by Robert V. Whitman.

	Specimen 1	Specimen 2	Specimen 3
Pore fluid	air	water	water
$\sigma_{3f} = \sigma_{30} = \sigma_{10}$	15 lb/in^2	15 lb/in^2	30 lb/in^2
$u_0 = u_f$	0	0	15 lb/in^2
$\bar{\sigma}_{3f} = \bar{\sigma}_{30} = \bar{\sigma}_{10}$	15 lb/in^2	15 lb/in^2	15 lb/in^2
σ_{1f}	60.0 lb/in^2	~~60~~ 58.3 lb/in^2	~~[illegible]~~ 73.5 lb/in^2
$\bar{\sigma}_{1f}$	60.0 lb/in^2	58.3 lb/in^2	58.5 lb/in^2
$\bar{\sigma}_{1f} / \bar{\sigma}_{3f}$	4.0	3.9	3.9



SAND SAMPLE

FIGURE 4

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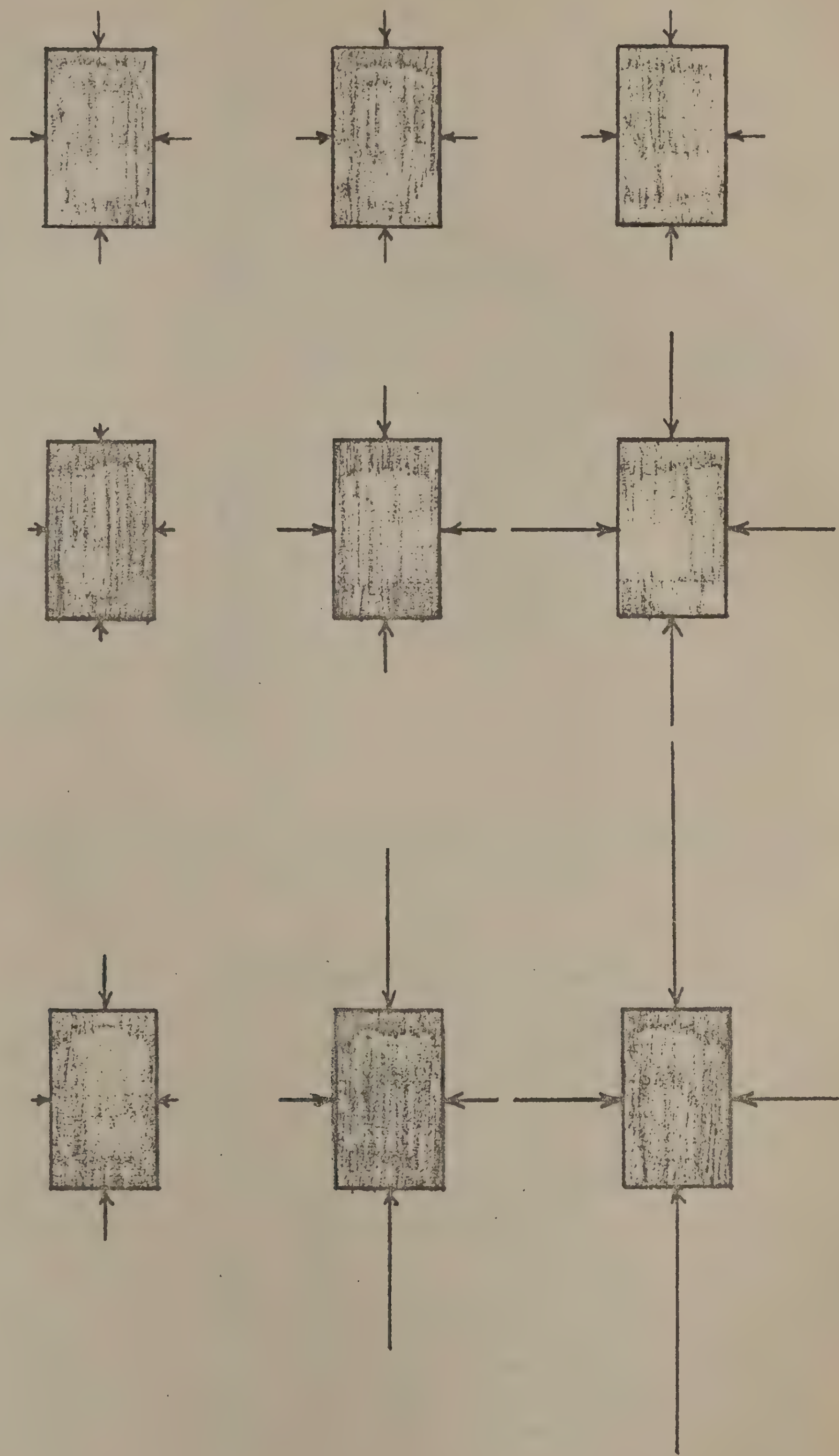

Step 1. Obtain several identical specimens; i.e. same water content, same stress history, same initial effective stress, etc.

Step. 2. Increase or decrease the confining pressure, allowing further consolidation. Volumes of samples will change.

Step 3. Apply axial stress, permitting consolidation. Axial stresses must be applied slowly. Volumes of samples will change.

FIGURE 5

PROGRAM OF CONSOLIDATED-DRAINED (CD) TRIAXIAL TESTS

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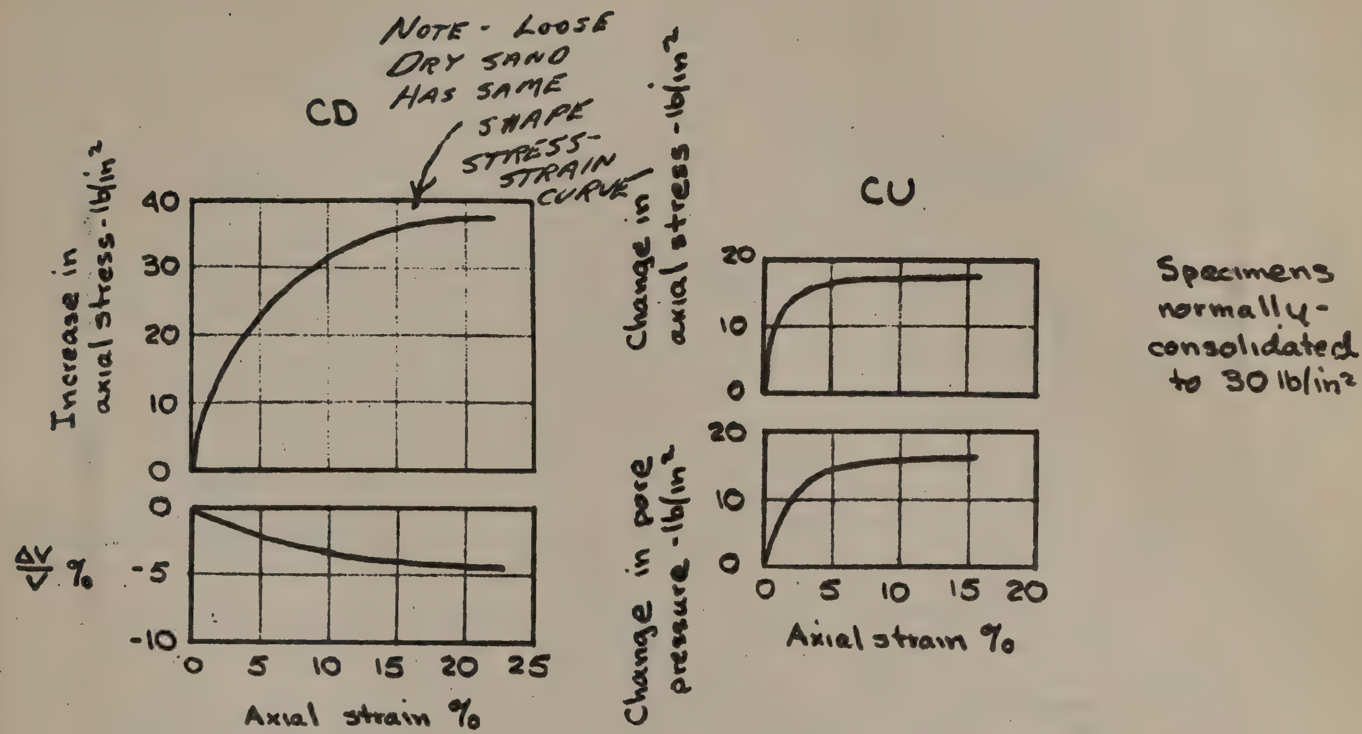

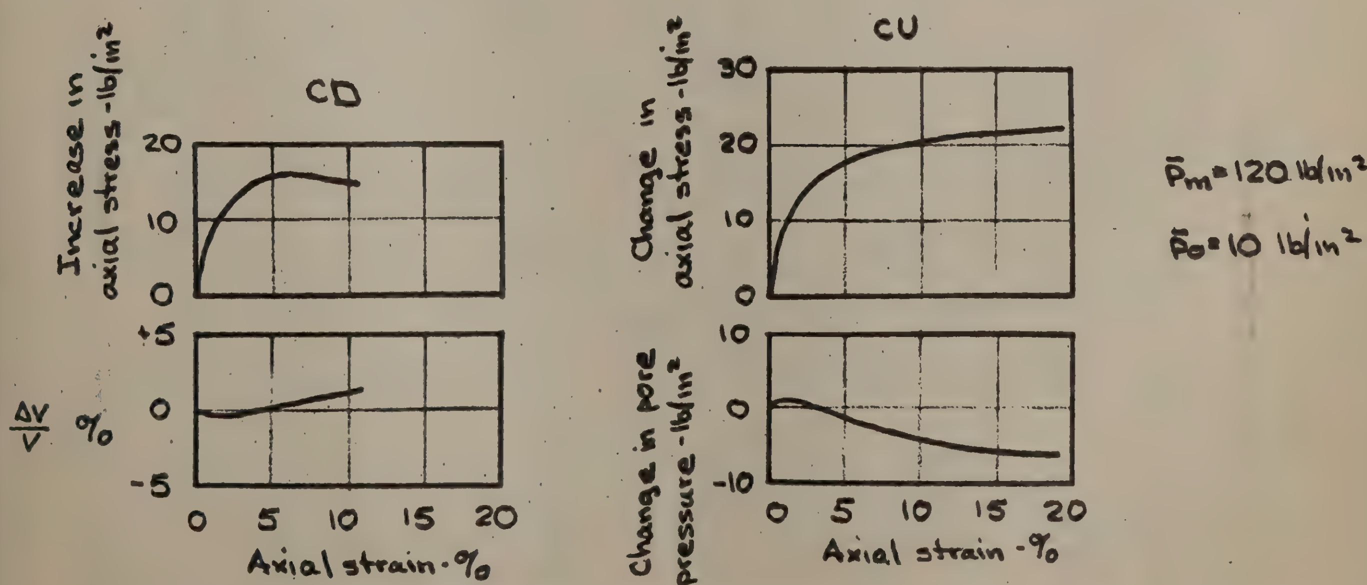

Note - Since there is no volume change, the strain necessary to fail a speciman is less in a CU test than in a CD test.

after PARRY (1960)

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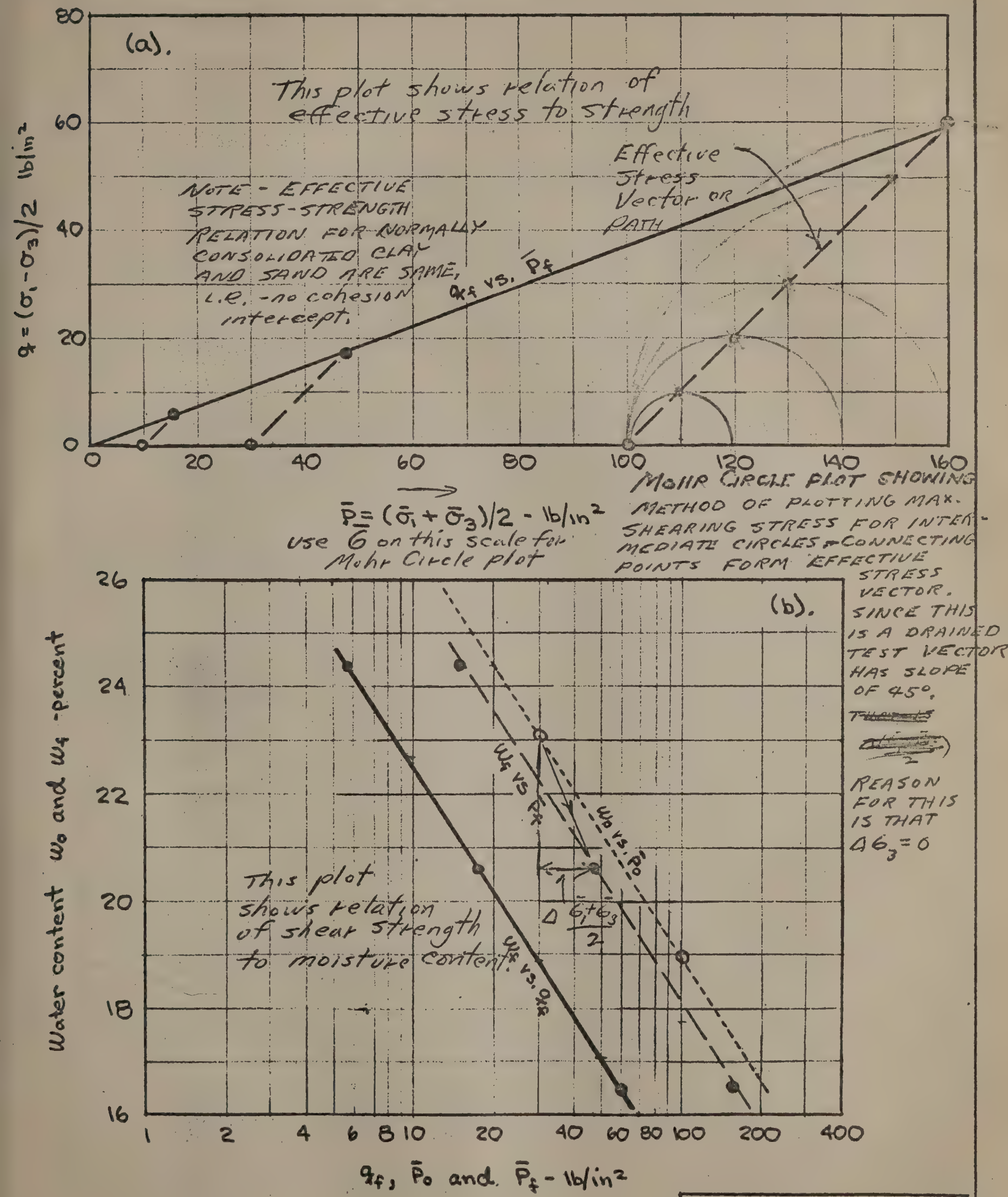
FIG. 6



Normally-consolidated
Weald clay
CD tests

FIGURE 7

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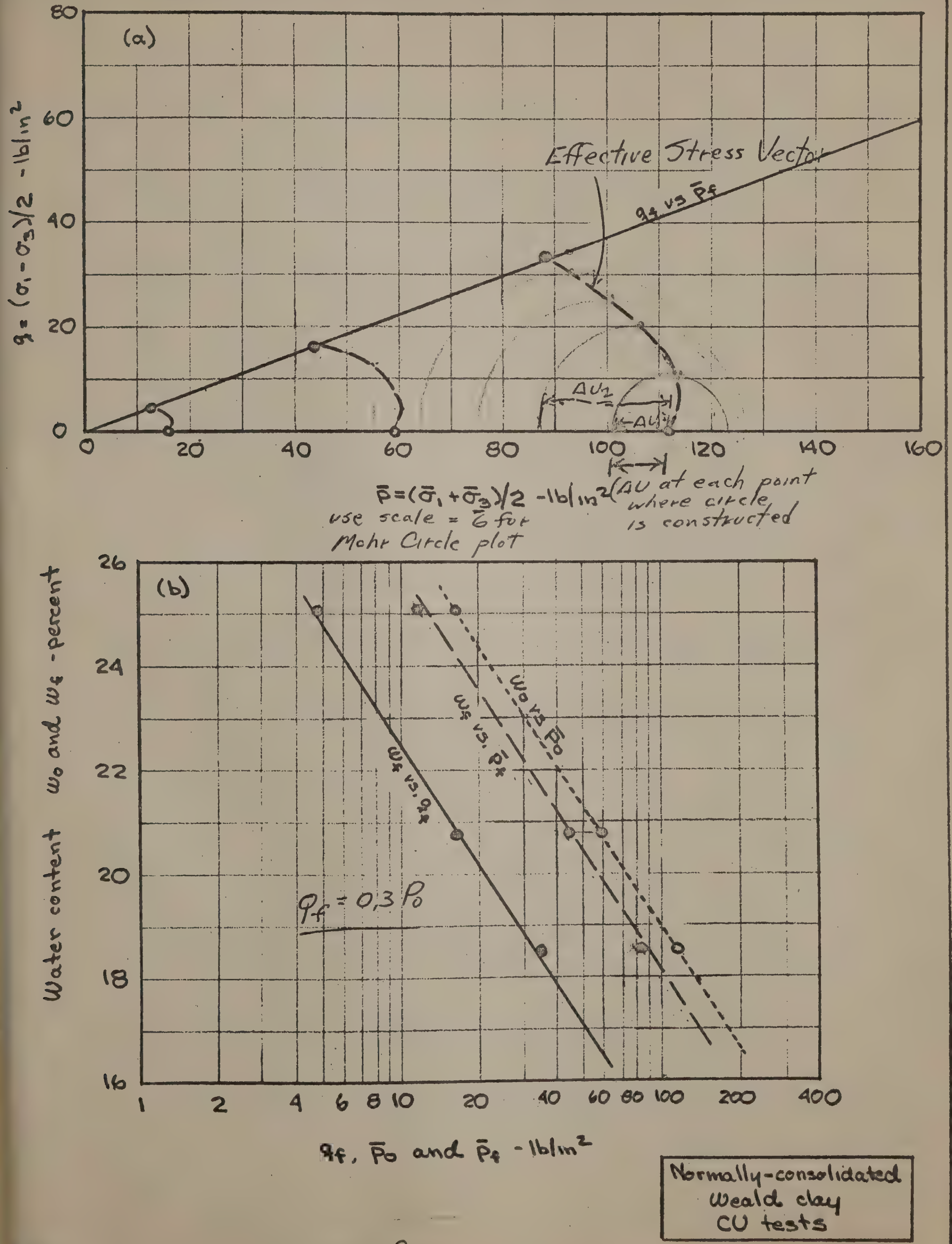

FIGURE 8

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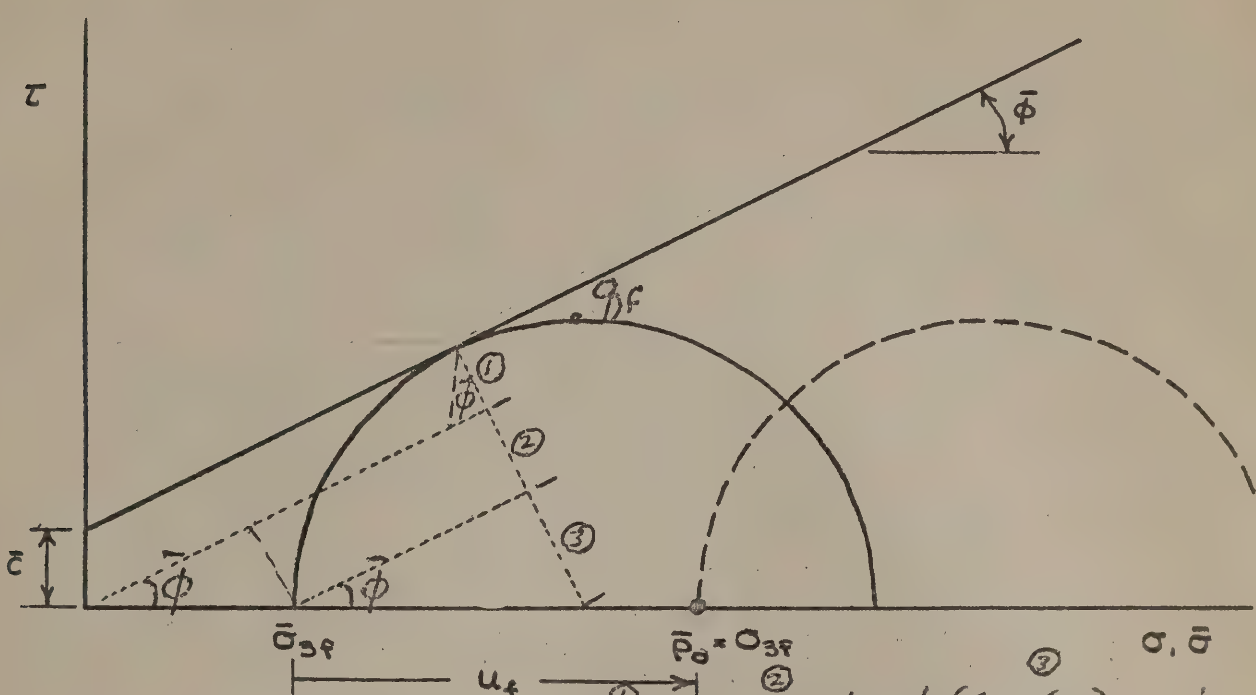

$$\frac{1}{2}(\sigma_{1f}-\sigma_{3f}) = \overset{①}{\bar{c}\cos\bar{\phi}} + \overset{②}{\bar{\sigma}_{3f}\sin\phi} + \overset{③}{\frac{1}{2}(\sigma_{1f}-\sigma_{3f})\sin\phi}$$

At failure: $\frac{1}{2}(\sigma_{1f}-\sigma_{3f}) = q_f = \dfrac{\bar{c}\cos\bar{\phi} + \bar{\sigma}_{3f}\sin\bar{\phi}}{1-\sin\bar{\phi}}$

If $\Delta u = A_f(\Delta\sigma_1 - \Delta\sigma_3)$, $\quad u_f = A_f(\sigma_{1f}-\sigma_{3f}) = 2A\,q_f$

$$q_f = \frac{\bar{c}\cos\bar{\phi} + (\bar{P}_o - 2A_f q_f)\sin\bar{\phi}}{1-\sin\bar{\phi}} \qquad \text{eq D-4-1}$$

If $\bar{c} = 0$ $\quad q_f/\bar{P}_o = \dfrac{\sin\bar{\phi}}{1+(2A_f-1)\sin\bar{\phi}}$ $\qquad$ eq D-4-2

if $A_f = 1$ $\quad q_f/\bar{P}_o = \dfrac{\sin\bar{\phi}}{1+\sin\bar{\phi}}$

if $A_f = 1/2$ $\quad q_f/\bar{P}_o = \sin\bar{\phi}$

DERIVATION OF RELATIONSHIP BETWEEN q_f, $\bar{P}_o$, $\bar{\phi}$ and PORE PRESSURE PARAMETER AT FAILURE (A_F)

FIGURE 9

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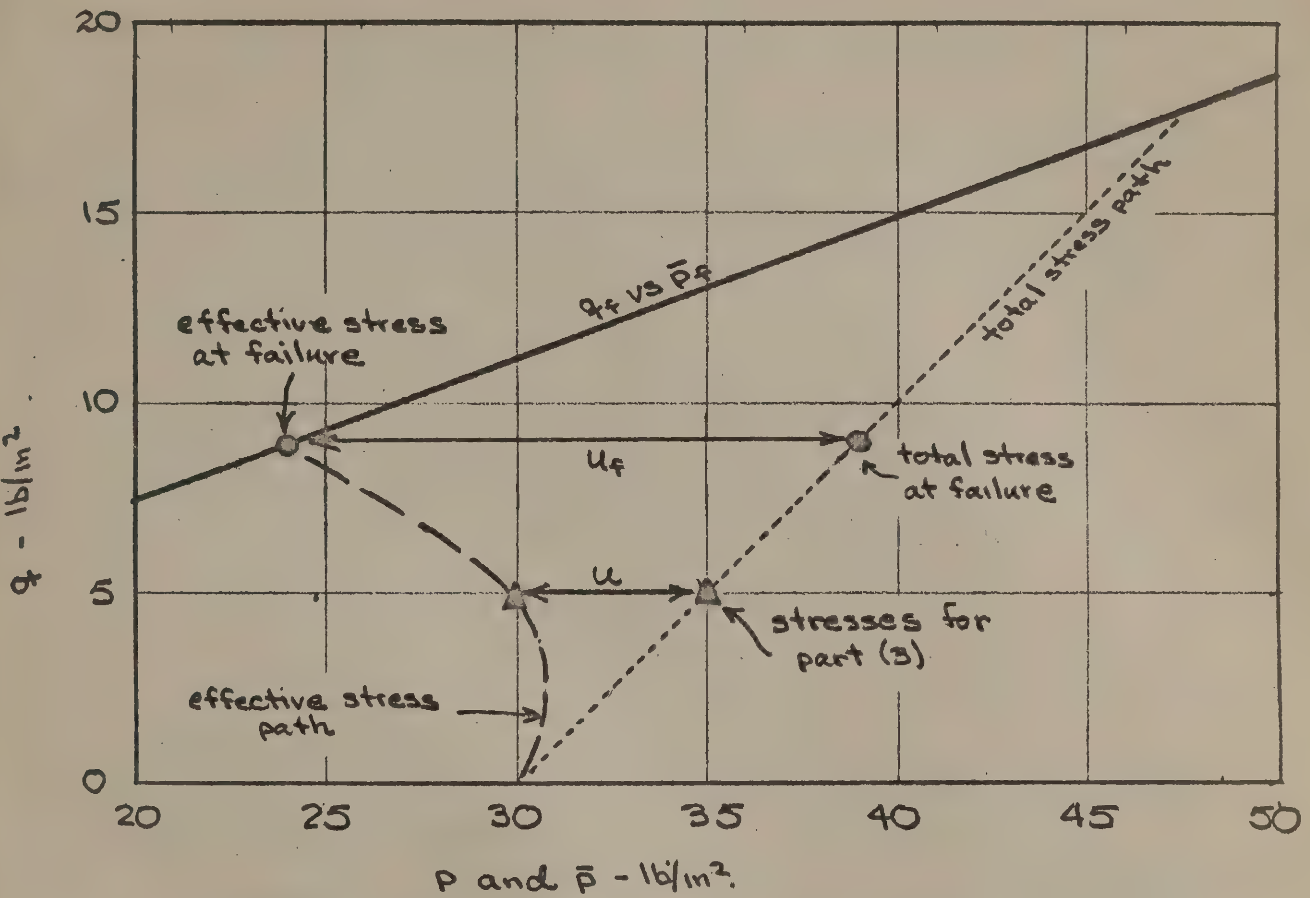
7/31/61

EXAMPLE D.3



(1). At failure: $p_f = 39.0$ lb/in², $\bar{p}_f = 24.0$ lb/in²

$$p_f - \bar{p}_f = \tfrac{1}{2}\left[(\sigma_{1f} - \bar{\sigma}_{1f}) + (\sigma_{3f} - \bar{\sigma}_{3f})\right] = u_f; \quad \boxed{u_f = 15 \text{ lb/in}^2}$$

(2). At failure: $\bar{p}_f = 24.0$ lb/in², $q_f = 9.0$ lb/in²

$$\bar{p}_f + q_f = \bar{\sigma}_{1f}; \quad \boxed{\bar{\sigma}_{1f} = 33 \text{ lb/in}^2, \ \bar{\sigma}_{3f} = 15 \text{ lb/in}^2}$$

Alternate solution: $\sigma_{3f} = \sigma_{30} = 30$ lb/in²; $\bar{\sigma}_{3f} = \sigma_{3f} - u_f = 15$ lb/in²

(3). Change in axial stress $= 2q = 40 - 30 = 10$ lb/in², $q = 5$ lb/in².

From graph, $u = 5$ lb/in² $\quad \boxed{\begin{array}{l}\bar{\sigma}_{1f} = 40-5 = 35 \text{ lb/in}^2 \\ \bar{\sigma}_{3f} = 30-5 = 25 \text{ lb/in}^2\end{array}}$

FIGURE 10

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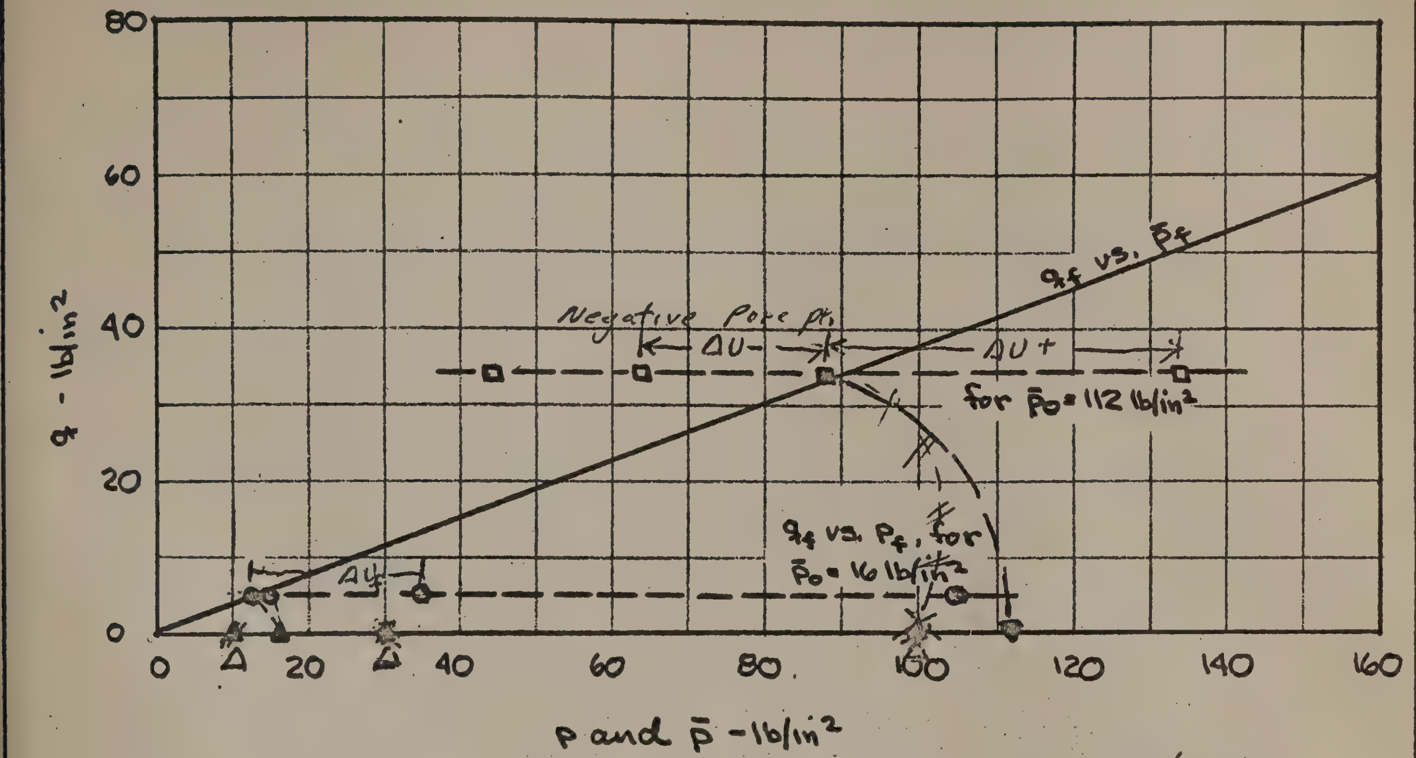

Note q_f vs. $\bar{p}_f$ same for CD, CU, and UU.

Initial stresses

σ_3=10,30,100 △ total confining stress

$\bar{\sigma}_3$=16 ▲ effective stress for specimens consolidated to 16 lb/in²

$\bar{\sigma}_3$=112 ▼ effective stress for specimens consolidated to 112 lb/in²

Stresses at failure

○ total, ● effective — for specimens consolidated to 16 lb/in²

□ total, ■ effective — for specimens consolidated to 112 lb/in²

Normally-consolidated Weald clay
UU tests

FIGURE 11

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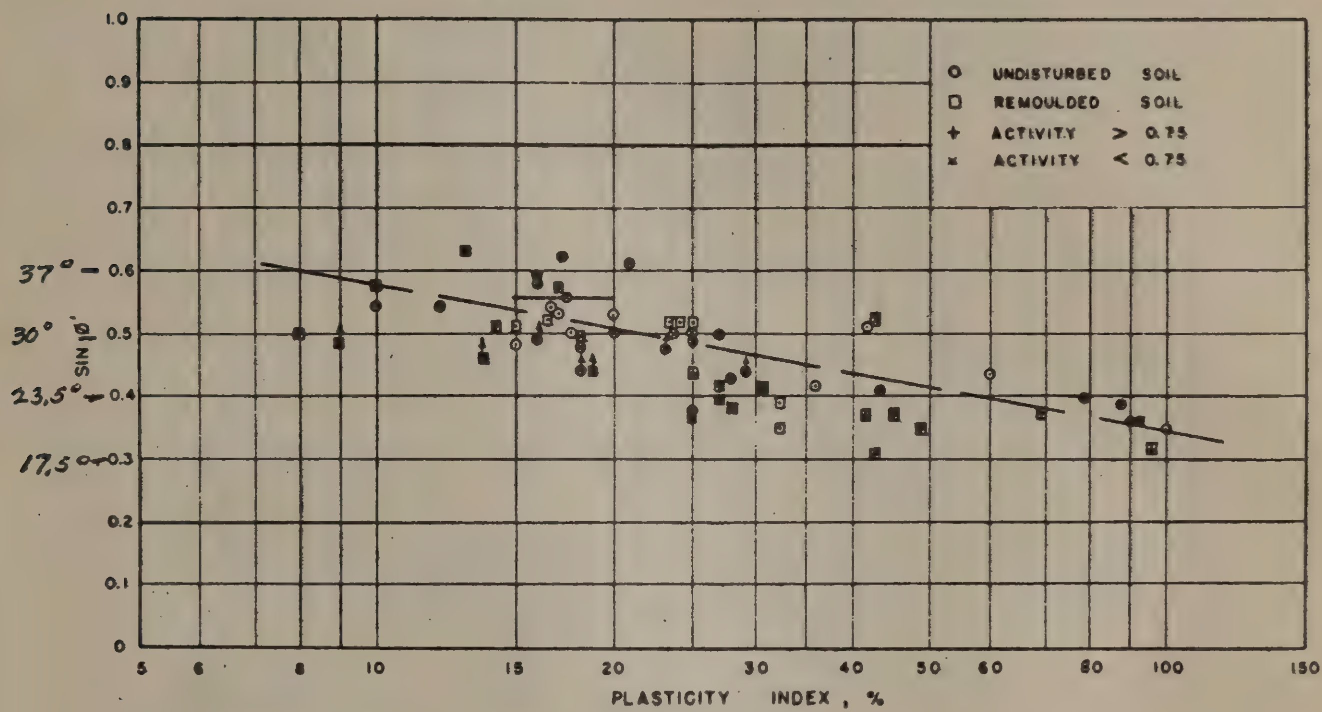
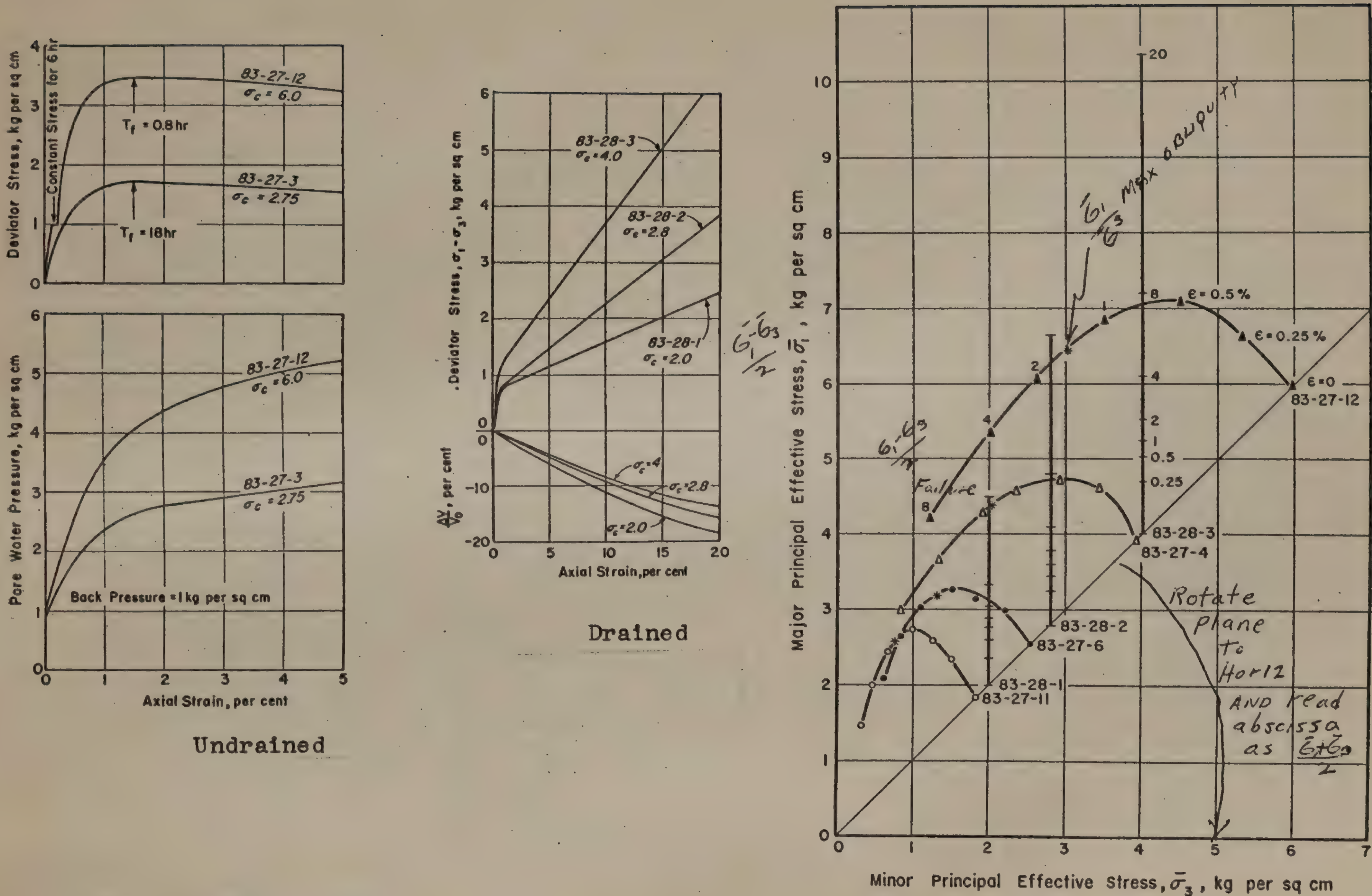


RELATIONSHIP BETWEEN SIN Ø' AND PLASTICITY INDEX

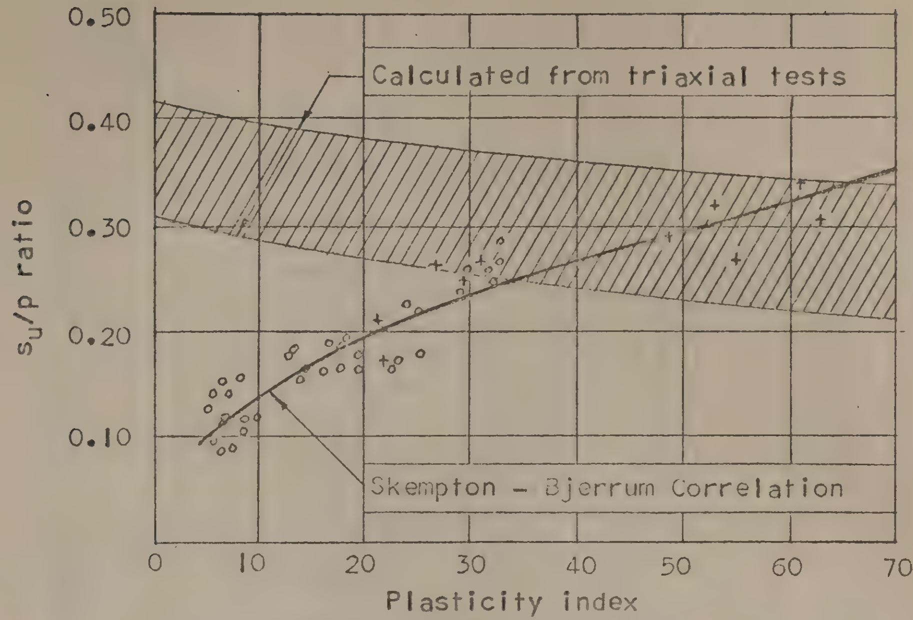
after KENNEY (1959)

FIGURE 12

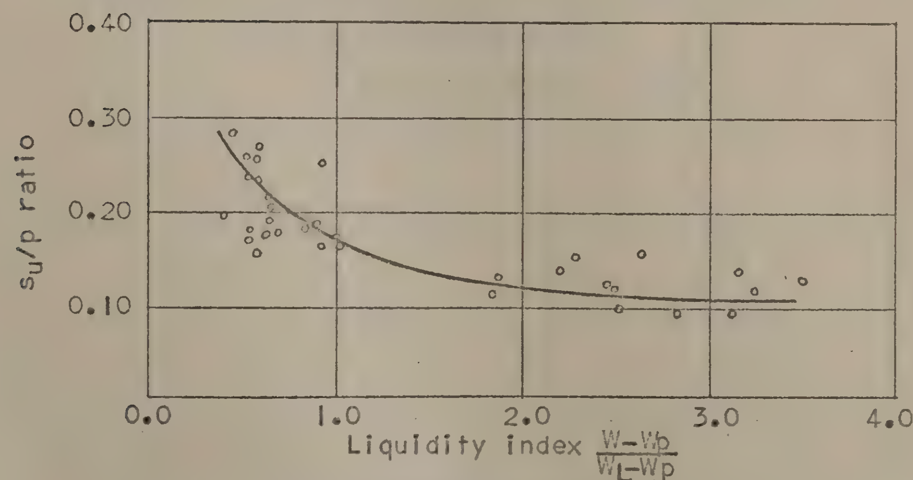


after CRAWFORD (1959)
KENNEY (1959)

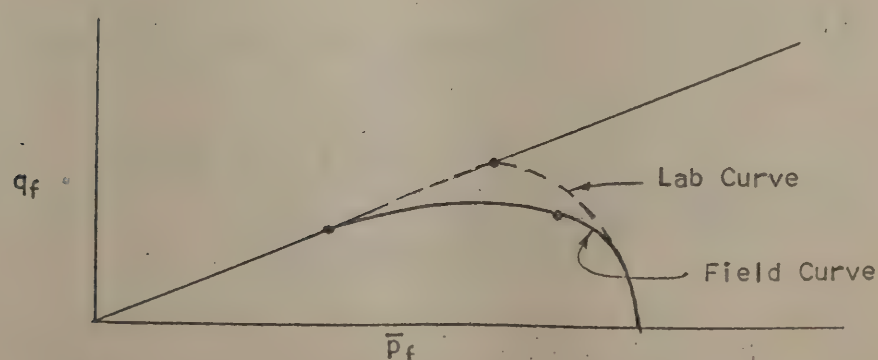
FIG. 13



Relationship Between s_u/p And Plasticity Index.

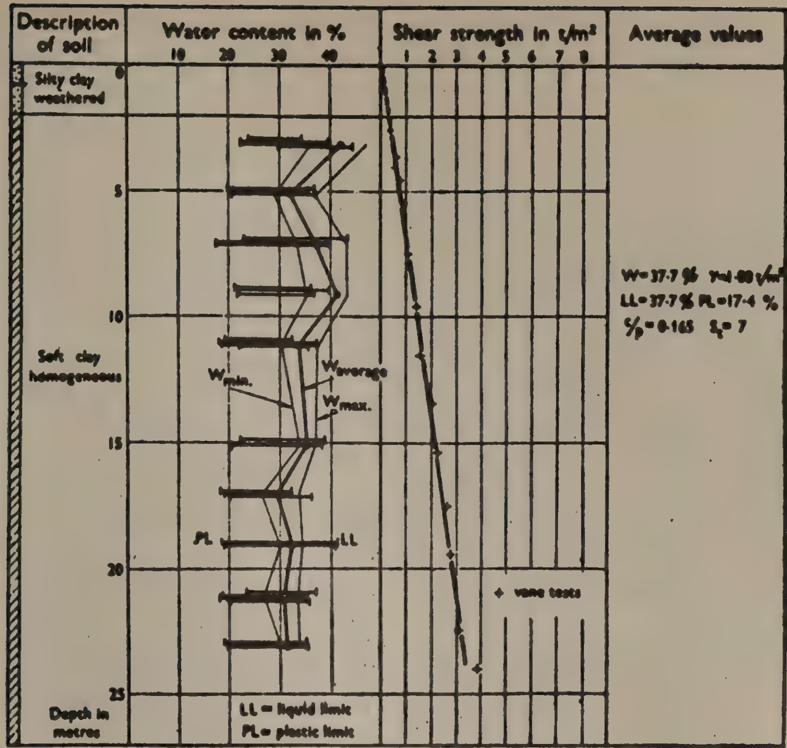


Relationship Between s_u/p And Liquidity Index.



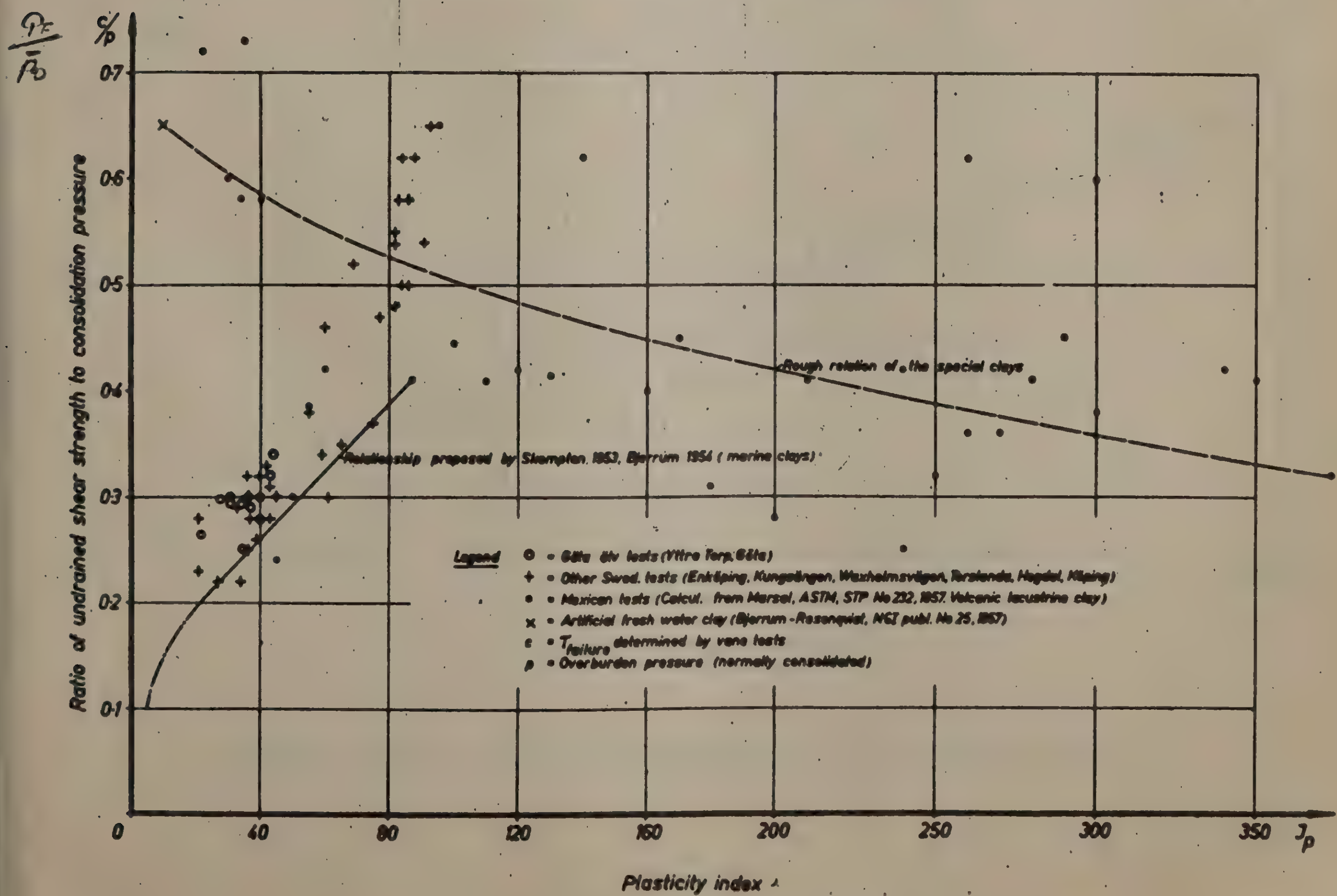
Effective Stress Diagram for Sensitive Clays

FIG. 14



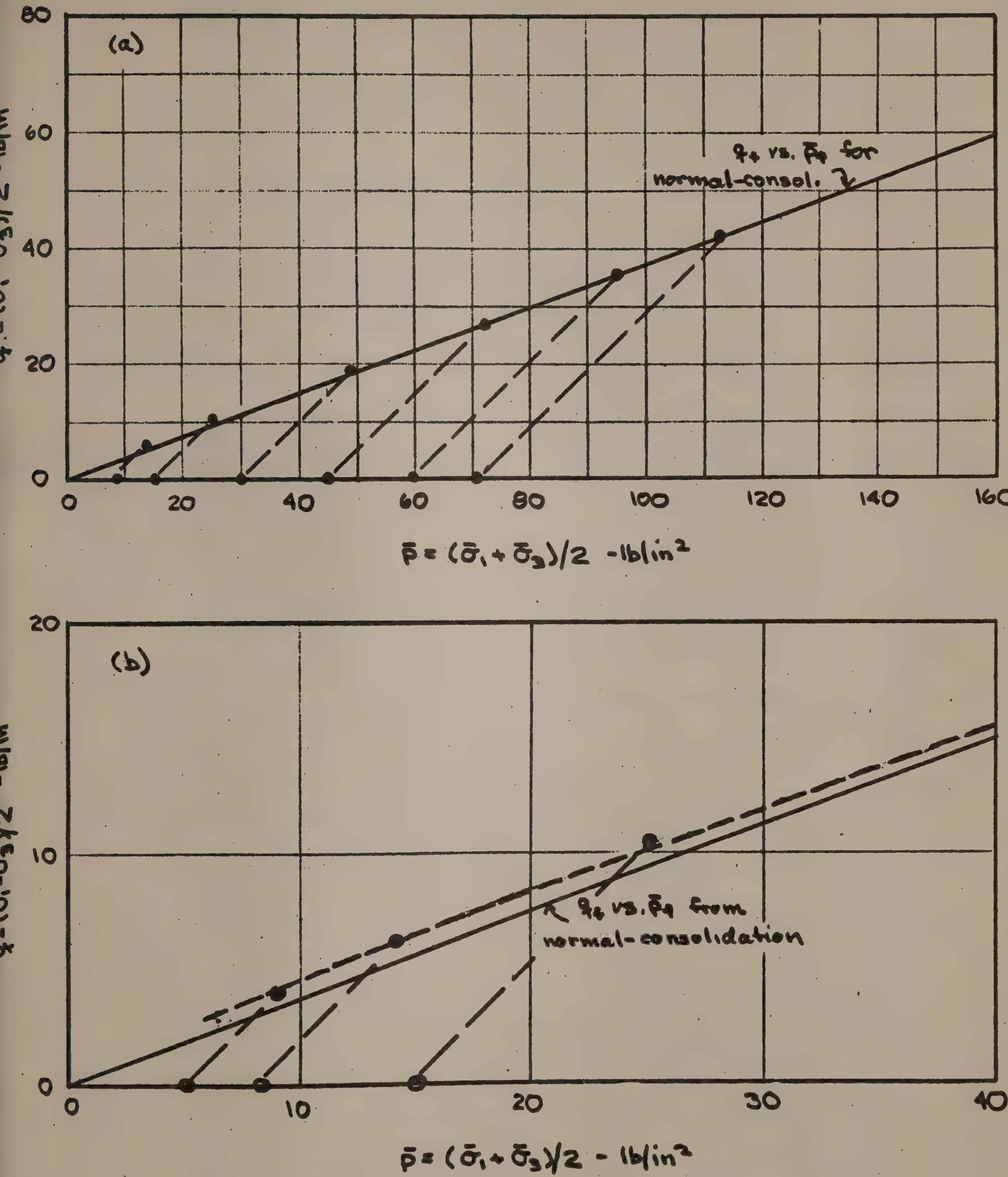
Results of a boring in Drammen

after BJERRUM (1954)



after OSTERMAN (1960)

FIGURE 13



Weald clay - overconsolidated to $\bar{p}_m = 120$ lb/in^2

FIGURE 16

RVW
8/1/61

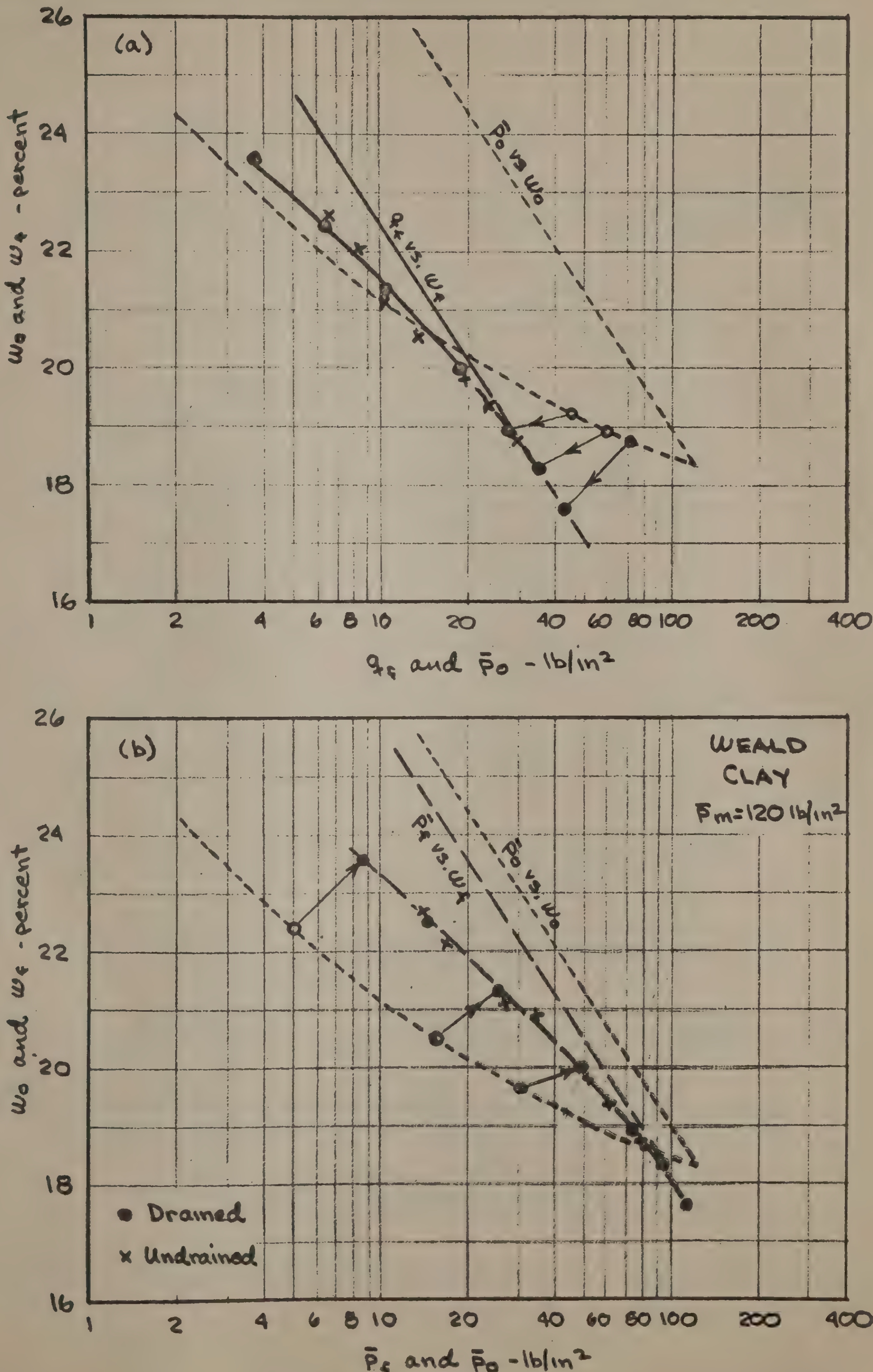
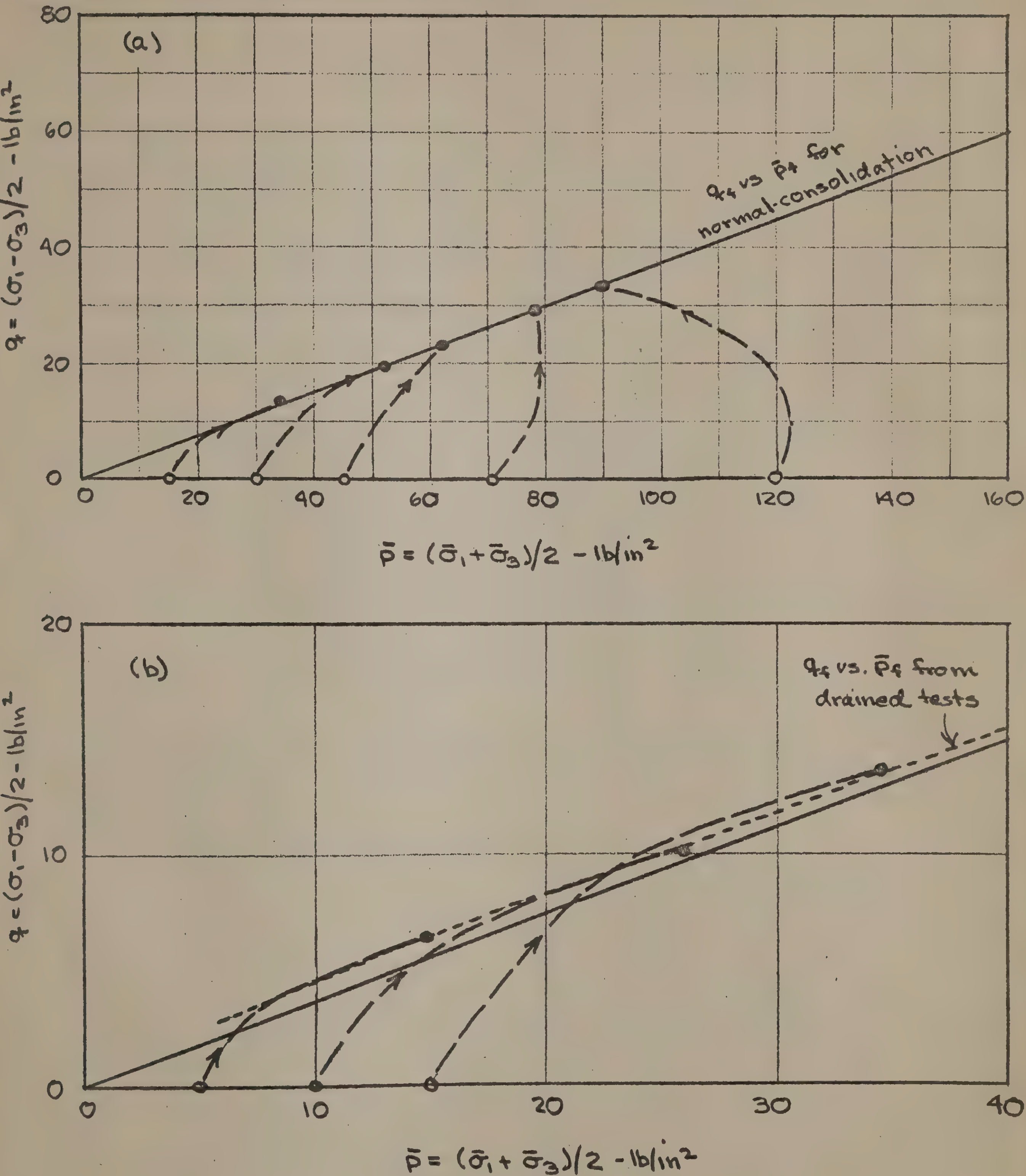


FIGURE 17

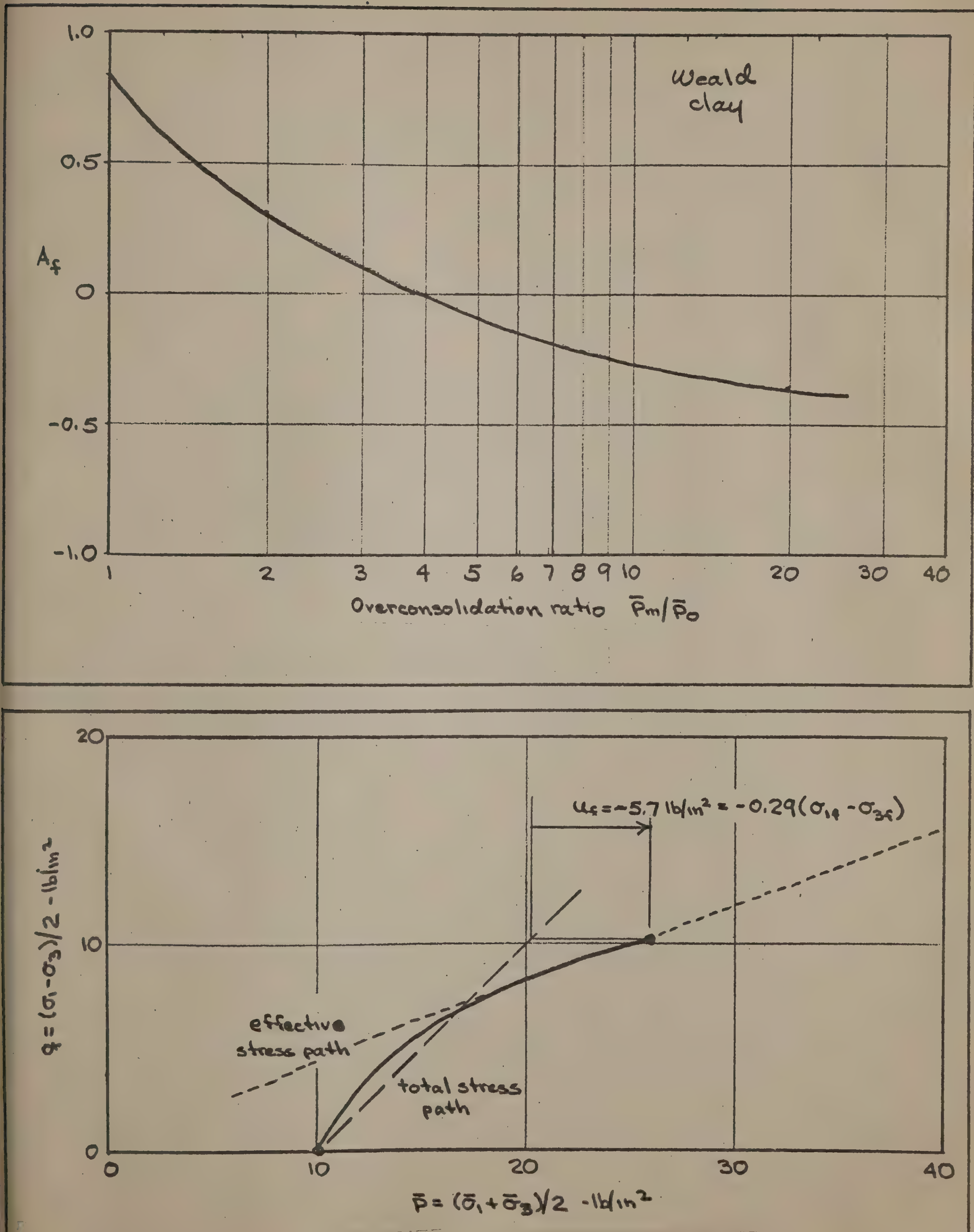
RVW
8/1/61



Weald clay - overconsolidated to $\bar{p}_m = 120$ lb/in².

FIGURE 18

RVW
8/1/61



RVW
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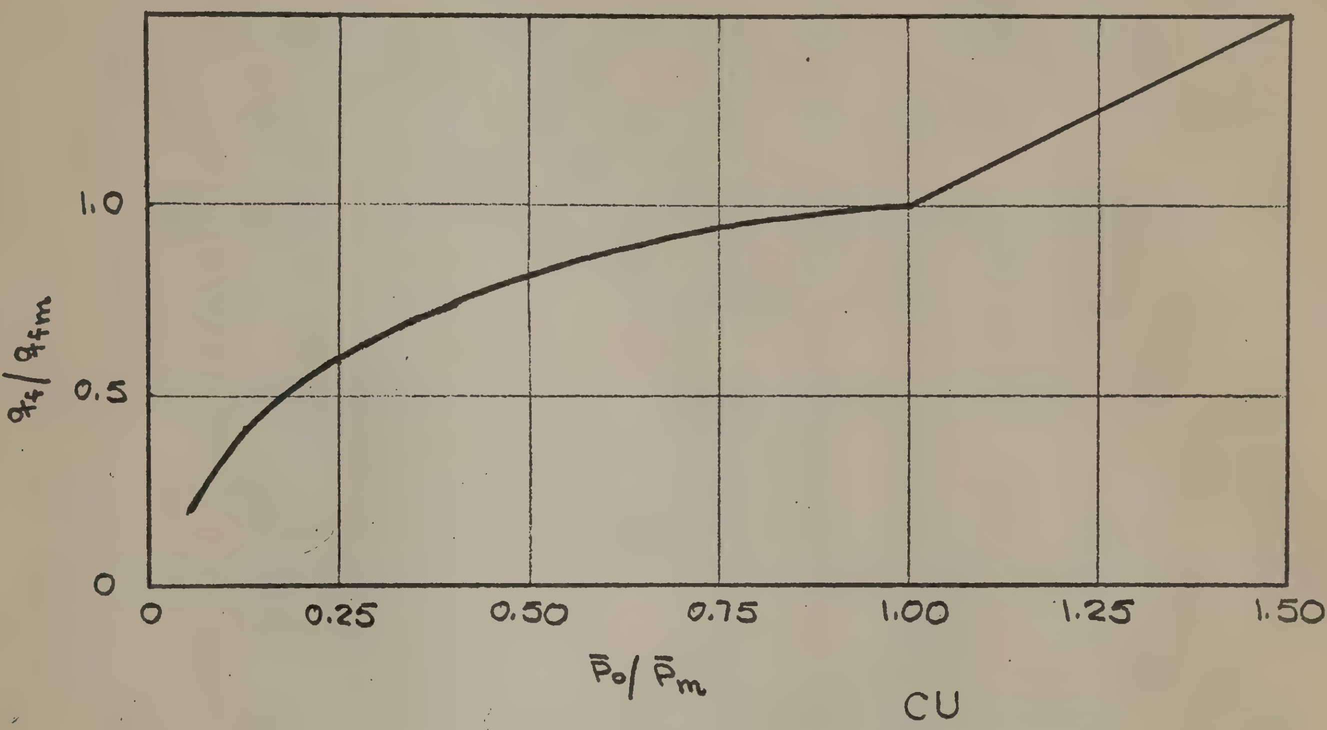
FIGURE 19



RVW
8/2/61

FIGURE 20

VII. MEASUREMENT OF SHEAR STRENGTH OF SATURATED COHESIVE SOILS

The methods of measuring strength and pore pressure in triaxial tests are presented in detail in "Triaxial Testing" by Bishop and Henckel and articles in the Boulder Colorado Shear Strength Conference proceedings.

Rough notes and outline of this subject are in detail course notes - Part C. Following are abstracted some of the more pertinent statements.

A. Field Vane Tests

1. Use rotation rate of 0.1° per sec.
2. Shear strength from vane shear test plotted on Mohr Circle is on standard envelope. To obtain q_f divide $\left(\frac{\sigma_1 - \sigma_3}{2}\right)$ vane shear strength by cos ϕ (estimated) $q_f = \frac{S_{v.s.}}{\cos.\phi}$.
3. Use vane shear test in soils with shear strength less than 1000 to 2000 PSF.
4. Torque in rods makes stress-strain plot inaccurate.

B. Test Results

1. Good sampling should give laboratory M.C. equal to field M.C.
2. Compare all test results on $\frac{c}{p}$ vs P.I. chart for consistency of results. We should put this into practice.
3. On triaxial tests plot vol. change vs time during consolidation. If there are leaks in membrane they will be obvious in ΔU plot. Leaks decrease strength.

VIII. TYPES OF STABILITY PROBLEMS

The detailed course notes contain a paper entitled slope stability analysis labeled part B. This should be read by anyone doing stability analysis. The main points of this section are as follows:

A. Construction with No Change in M.C.

When clays are loaded or unloaded in a manner where no change in moisture content occurs in construction then CU test results may be used in the same type of analysis we use. This applies to 75% to 90% of the problems encountered.

B. Stability of Clay Cuts

1. Pore Pressures

The strength of clay depends on the effective stresses. These stresses can change by ground water flow and seasonal water table variations.

2. Non-uniformity of Deposit

In a slide at Jackfield, England the failure occurred on a clay layer <u>two</u> inches thick with M.C. 10% higher than adjacent clay.

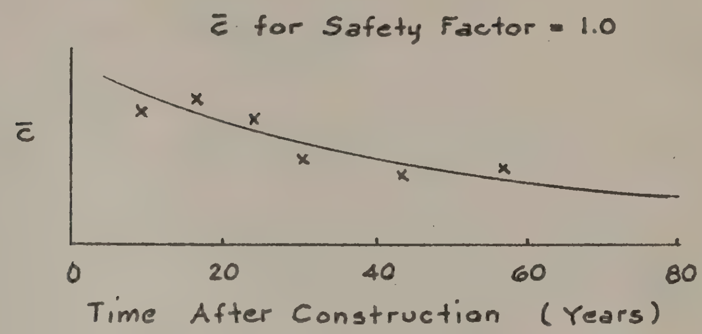
3. Rate of Strain

Laboratory testing indicates the C intercept of a clay decreases as the rate of shear decreases.

4. English RR Analysis

Many slides were analyzed on RR constructed 80 years ago with a well-documented history of the slides. The following curve summarizing their

stability analysis explains time effects on clay strength.



5. Use of CU, UU, and U Tests for Cut Stability

The undrained shear strength was used to analyze cut slides with following results.

SF = 1.9 and higher for overconsolidated clays

SF = 0.6 to 1.2 for normally consolidated clays

6. Effect of Permeable Layers in Clay

See article in Geotechnique vol. 5 by Ward, Penman & Gibson.

7. Taylor's charts are being revised to account for pore pressure - See Geotechnique - December, 1960.

C. Effective Stress Analysis

1. Where Used

a. Embankment problems where construction depends on increase of shear strength with consolidation.

b. Cut problems where strength changes due to GW variation and relief of load may be measured to forecast approach of instability.

2. Required Information for Analysis

a. Total stresses

b. From laboratory determine strength as a function of effective stress and pore pressure parameters

c. Field pore pressures

3. Difficulties

Pore pressures in the field and in the laboratory sample are a function of shearing strain and volume change due to consolidation (Terzhagi Theory). The pore pressure is a sum of the two. Each can be analyzed individually by theoretical means but nobody knows how it works in the field. See Figure 22 (Pg.46) which summarizes all information on settlement determined by effective stress.

4. The Unanswered Question

Do laboratory pore pressures duplicate field pore pressures? Since pore pressure is a function of strain rates, lateral pressure in the ground, water temperature and rotation of principal stresses along failure arc, it is easy to see the question is unanswered. MIT is conducting two oil tank stability projects from which they hope to get field data to compare with laboratory data.

5. Future Application

The effective stress analysis method is just being tried on field stability problems. There is a great need for well instrumented field projects where sufficient data can be obtained to tie theory with field performance.

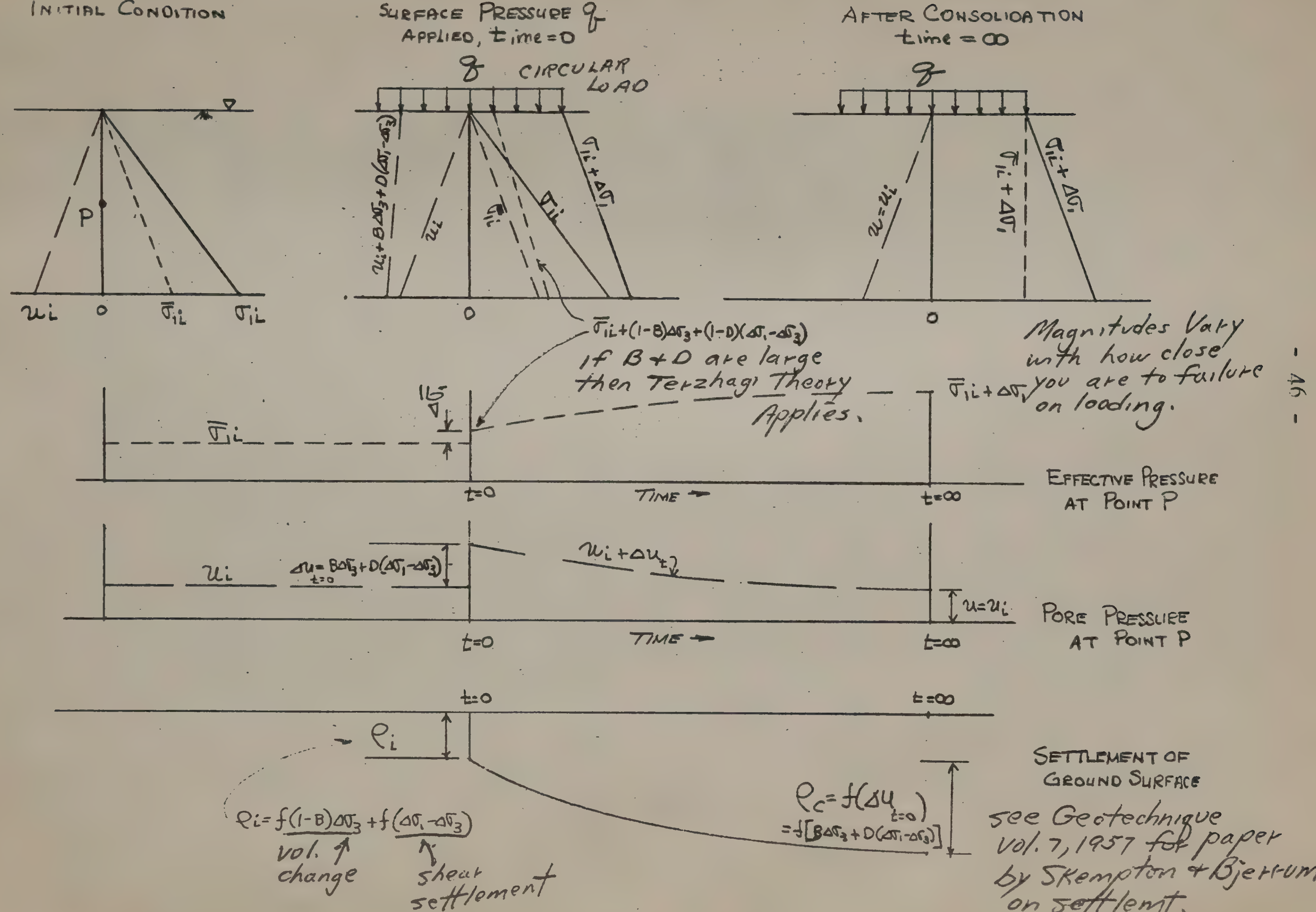

INITIAL CONDITION
SURFACE PRESSURE q APPLIED, time = 0
AFTER CONSOLIDATION time = ∞
q CIRCULAR LOAD
P
u_i 0 $\bar{\sigma}_{1i}$ σ_{1i}
$u_i + B\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3)$
$\sigma_{1i} + \Delta\sigma_1$
$u = u_i$
$\bar{\sigma}_{1i} + \Delta\sigma_1$
$\bar{\sigma}_{1i} + (1-B)\Delta\sigma_3 + (1-D)(\Delta\sigma_1 - \Delta\sigma_3)$
If B + D are large then Terzhagi Theory Applies.
Magnitudes Vary with how close you are to failure on loading.
EFFECTIVE PRESSURE AT POINT P
TIME →
t=0
t=∞
$u_i + \Delta u_t$
$\Delta u_{t=0} = B\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3)$
PORE PRESSURE AT POINT P
SETTLEMENT OF GROUND SURFACE
ρ_i
$\rho_i = f(1-B)\Delta\sigma_3 + f(\Delta\sigma_1 - \Delta\sigma_3)$
vol. change
shear settlement
$\rho_c = f(\Delta u_{t=0})$
$= f[B\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3)]$
see Geotechnique Vol. 7, 1957 for paper by Skempton + Bjerrum on settlemt.

FIG. 22

6. Pore Pressure at Failure

The intense shearing action at failure will cause a decrease in pore pressure. Our experience at Weedsport confirms this as the pore pressure in the failure zone dropped from 10 PSI to 3 PSI at a constant rate over a seven day period prior to the failure.

D. Grand Summary of Stability Analysis

If the problem involves the shear of a saturated clay mass with no time for migration of pore fluid during the construction period, the problem is fairly straightforward. The method of analysis is simple, and it is not too difficult a matter to estimate the available shear strength.

If the problem is a true long-term stability problem in which failure results from a gradual decrease in the shear strength, the problem is also relatively straightforward, the method of analysis is somewhat more complex, and it is somewhat more of a job to obtain data upon which to base estimates of the available shear strength.

If neither of these situations exist, you really have a problem. The pore pressures at failure will depend partially upon natural ground water conditions, loading conditions and partially upon pore pressures set up during rapid shear distortions; and you don't know how to pick the proper pore pressure unless you know at exactly what point in its history the slope will fail. The

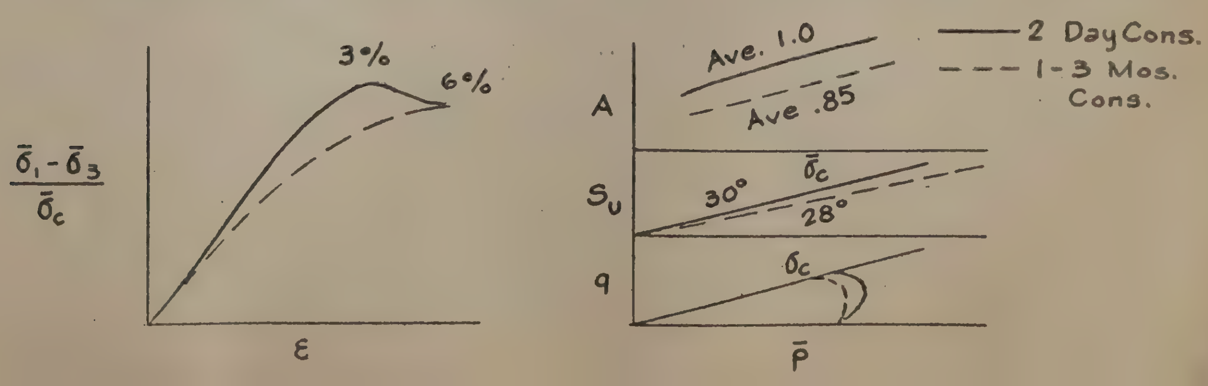
stability of compacted embankments falls in this category. Depending upon the magnitude of the project, one of several courses of action may be selected.

1. If the project is an important one, use the method of analysis together with the effective stress envelope. It will be necessary to use considerable judgment in the selection of the pore pressures to be used in the analysis. The actual pore pressures should be measured in the field and considerable judgment used in the application of these pore pressures to stability calculations.
2. For problems of less importance (and this means most highway embankments), use the unconsolidated, undrained shear strength of the compacted soil under confining pressures typical of those expected in the embankment. Include in the analysis a safety factor to allow for the possibility that the strength may be less than the value thus estimated. The simple methods of analysis used for undrained shear problems may be employed.

IX. DEVIATIONS FROM IDEALIZED STRENGTH BEHAVIOR

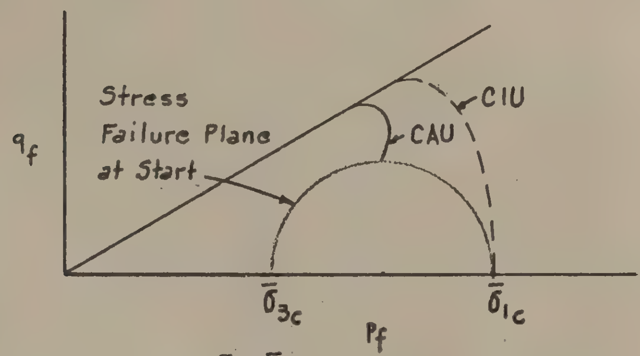
A. Effect of Test Consolidation Time

The results on a remolded clay for 2 day consolidation and one to three month consolidation indicated the following:



B. Anisotropic Consolidation

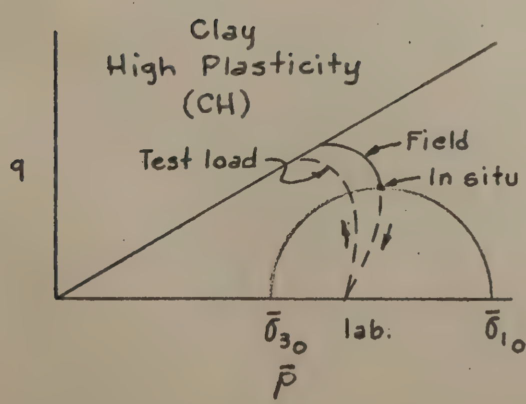
In the ground the stresses on a soil on all sides usually are not equal. The lateral effective stress may be 0.4 to 1.0 of the vertical effective stress. The ratio of $\frac{\bar{\sigma}_{3c}}{\bar{\sigma}_{1c}}$ called K_o may be 0.4 for a granular soil to 1.0 for a soft plastic soil. The stress vector for a consolidated undrained test with anisotropic consolidation is as follows: The test is labeled CAU.

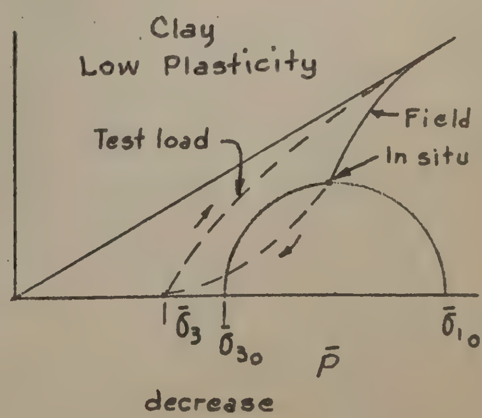


The decrease in $\frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2}$ for anisotropic consolidated samples compared to isotropic consolidation is 5 to 10%.

C. Stresses from Field Sampling

The stress history is shown as follows:





SUMMARY OF FACTORS AFFECTING STRENGTH BEHAVIOR

Factors Affecting S_u Behavior

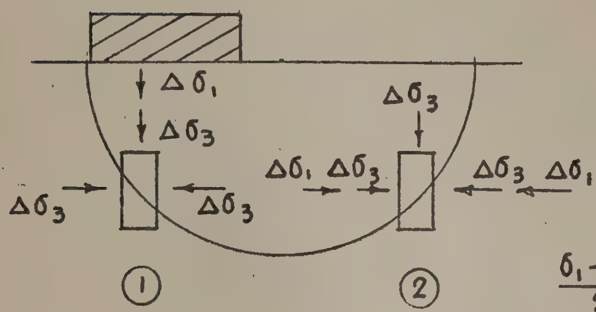
Normally Consolidated Clays

Factor	$\frac{Su}{\bar{\sigma}_c}$	Qu at $(\sigma_1-\sigma_3)$ Max.	A_f	ε_f	Remarks
1. Time of Consolidation	Inc. 5 to 10% log T_c	Decrease Several Degrees	Decrease	Decrease	Limited Data
2. Anisotropic K_o	Decrease 0-Fat 20%-Lean	Decrease 5-10° →	Usually Decrease	Decrease	Depends on Soil Type
3. Compression vs. Extension	10-25% Decrease for Extension	Constant	Ext. Increase	?	Fair Amount Data
4. Remoulding CU @ $\bar{\sigma}$ in situ	0 → 20 to 50% increase. Lower the Rem. W.C. the higher the strength increase	Probably Increase	Decrease	Increase	Limited to Date
5. Decreasing Strain Rate	10± 5% Dec/log Tf With thixotropic properties may be increase	Decrease	Increase	?	"

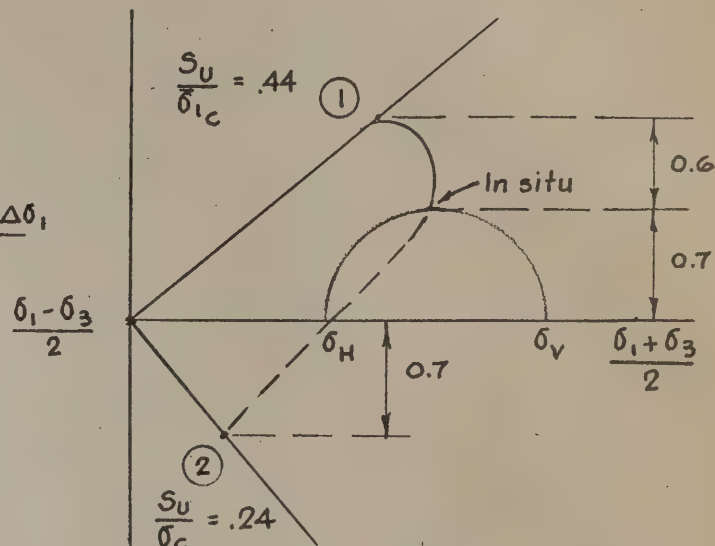
FIG. 21A

D. Reorientation of Principal Stresses

This is one of the most difficult factors to rationalize in applying the concept of shear strength along a circular arc failure. The change of stress direction is shown in Sketch A.



SKETCH A



Stability analysis using undrained strength works but the experts don't know why.

E. Summary of Factors Affecting Strength Behavior

(Normally Consolidated Clays) See Figure 21A

X. CASE HISTORIES

A. Refinery Tank - Kawasacki, Japan

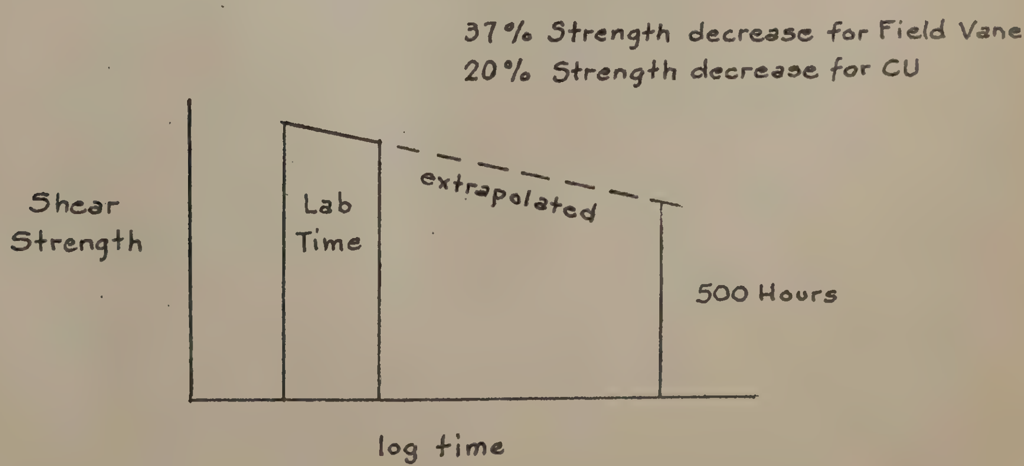
Soil profile and stress conditions are shown on Figure 23, 24 and 25. (pgs. 53, 54, & 55)

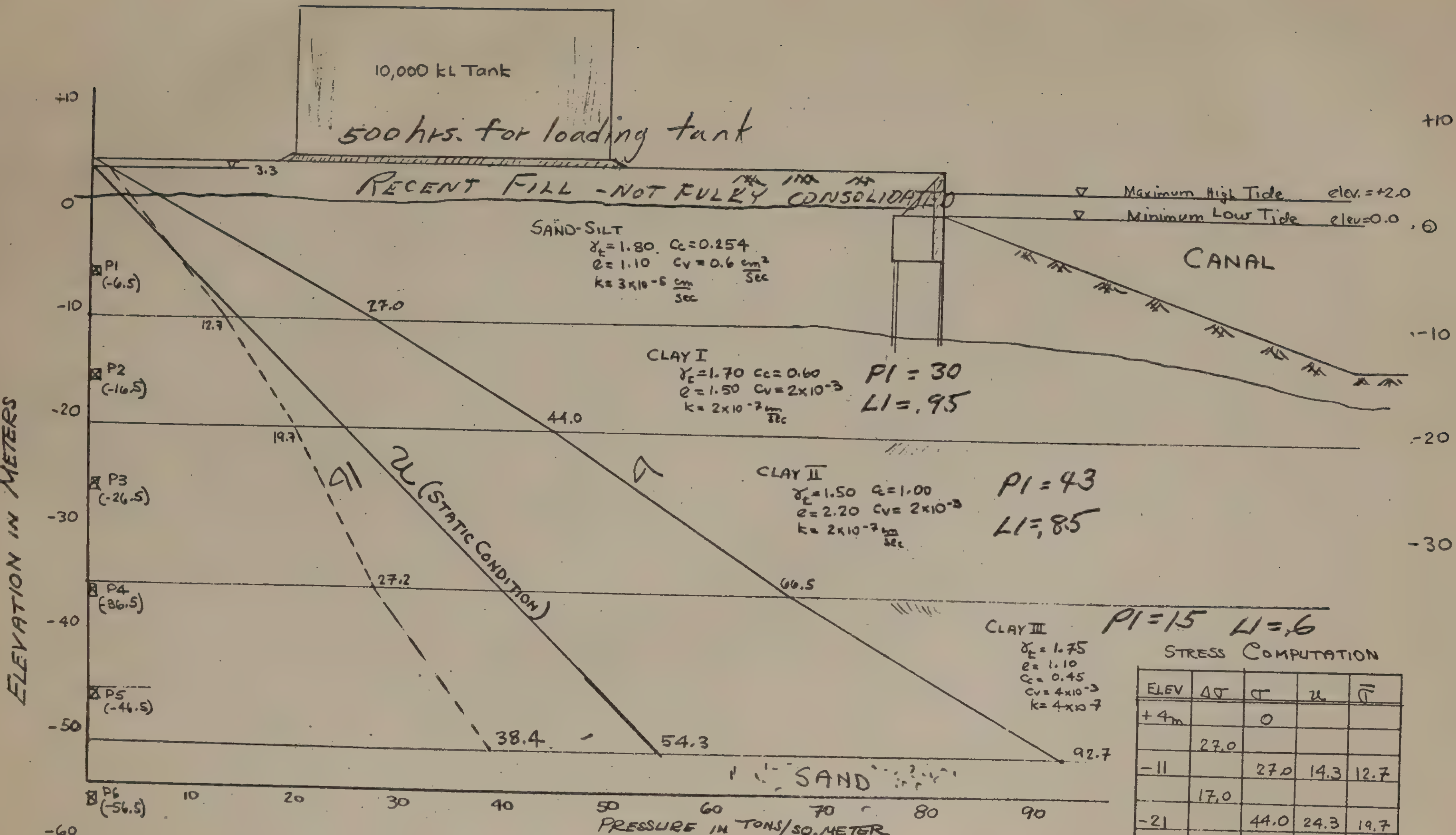
1. Determination of Shear Strength

Layer 2 was critical for stability and all the testing was tabulated in the following manner:

Ratio Based On	$\frac{S_u}{\bar{\sigma}_{1c}}$	
Field Vane	0.44	
F.V. corr. for strain rate (-37%)	0.28	Sensitivity
Lab Vane	0.31	10-20
Lab UU	0.30	
Unconfined	0.34	
$\overline{\text{CIU}}$	0.45	Lab M.C.
$\overline{\text{CAU}}$	0.43	6-10% lower than in situ.
$\overline{\text{CAUU}}$	0.40	
PI Chart	0.23	
CU M.C. vs. log S_s	0.34	
M.C. log S_s corrected for strain rate	0.27	

2. There was concern over the rate of laboratory loading vs field loading (500 hrs.) and the decrease in strength with slower strain. Three UU tests and 3 CD tests were failed over a period of five hours to find the decrease in strength with a slower strain rate. The results are shown as follows:





STRESS COMPUTATION

ELEV	$\Delta\sigma$	σ	u	$\bar{\sigma}$
+4m		0		
	27.0			
-11		27.0	14.3	12.7
	17.0			
-21		44.0	24.3	19.7
	22.5			
-36		66.5	39.3	27.2
	26.2			
-51		92.7	54.3	38.4

REFINERY SUBSOIL SECTION

KAWASAKI, JAPAN

FIG. 23

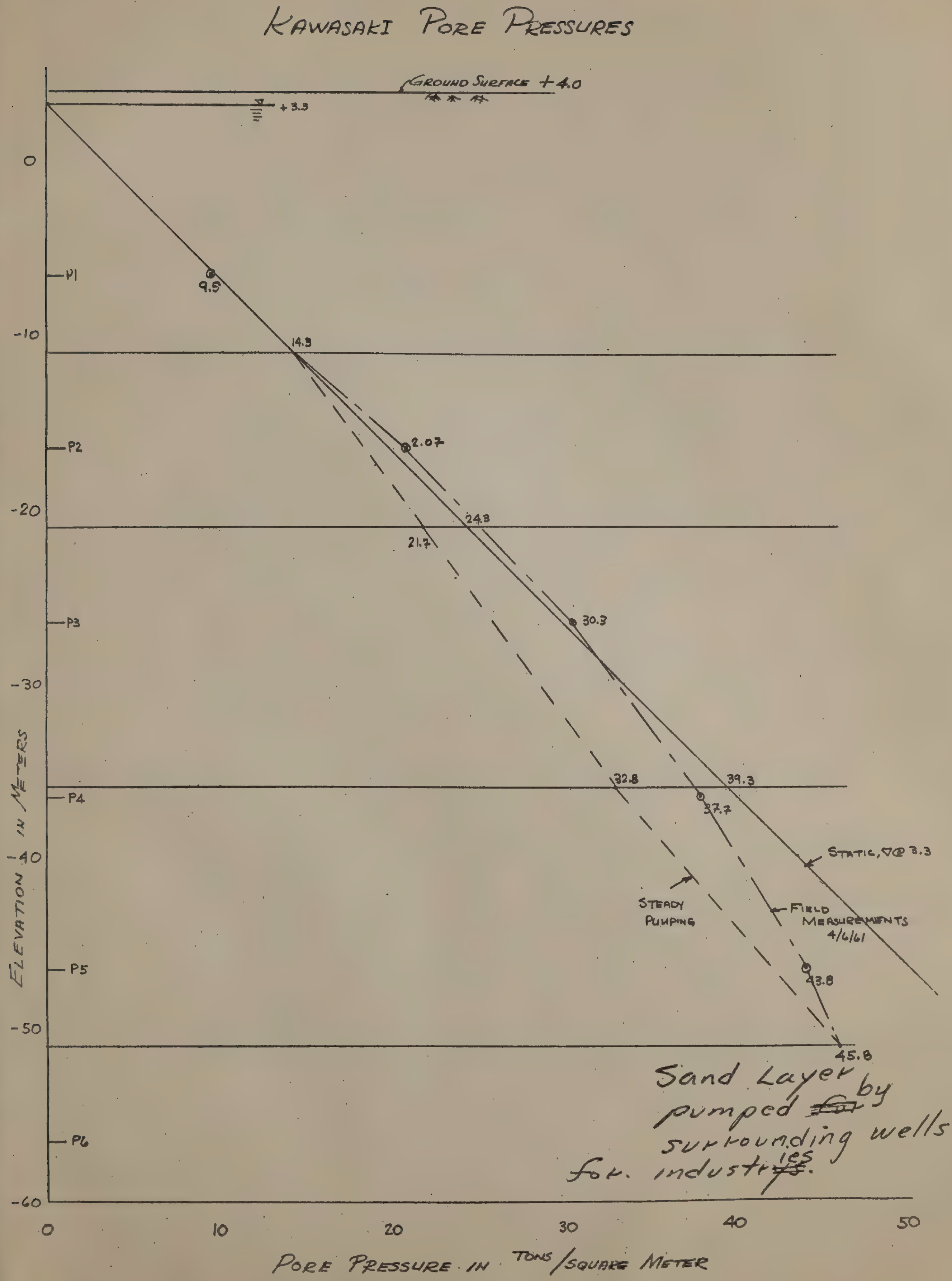

Kawasaki Pore Pressures
Ground Surface +4.0
+3.3
0
-10
-20
-30
-40
-50
-60
P1
P2
P3
P4
P5
P6
9.5
14.3
2.07
24.3
21.7
30.3
32.8
39.3
37.7
43.8
45.8
Static, ▽@ 3.3
Steady Pumping
Field Measurements 4/6/61
Sand Layer pumped by surrounding wells for industries.
0
10
20
30
40
50
Elevation in Meters
Pore Pressure in Tons/Square Meter

Fig. 24

KAWASAKI EFFECTIVE STRESSES

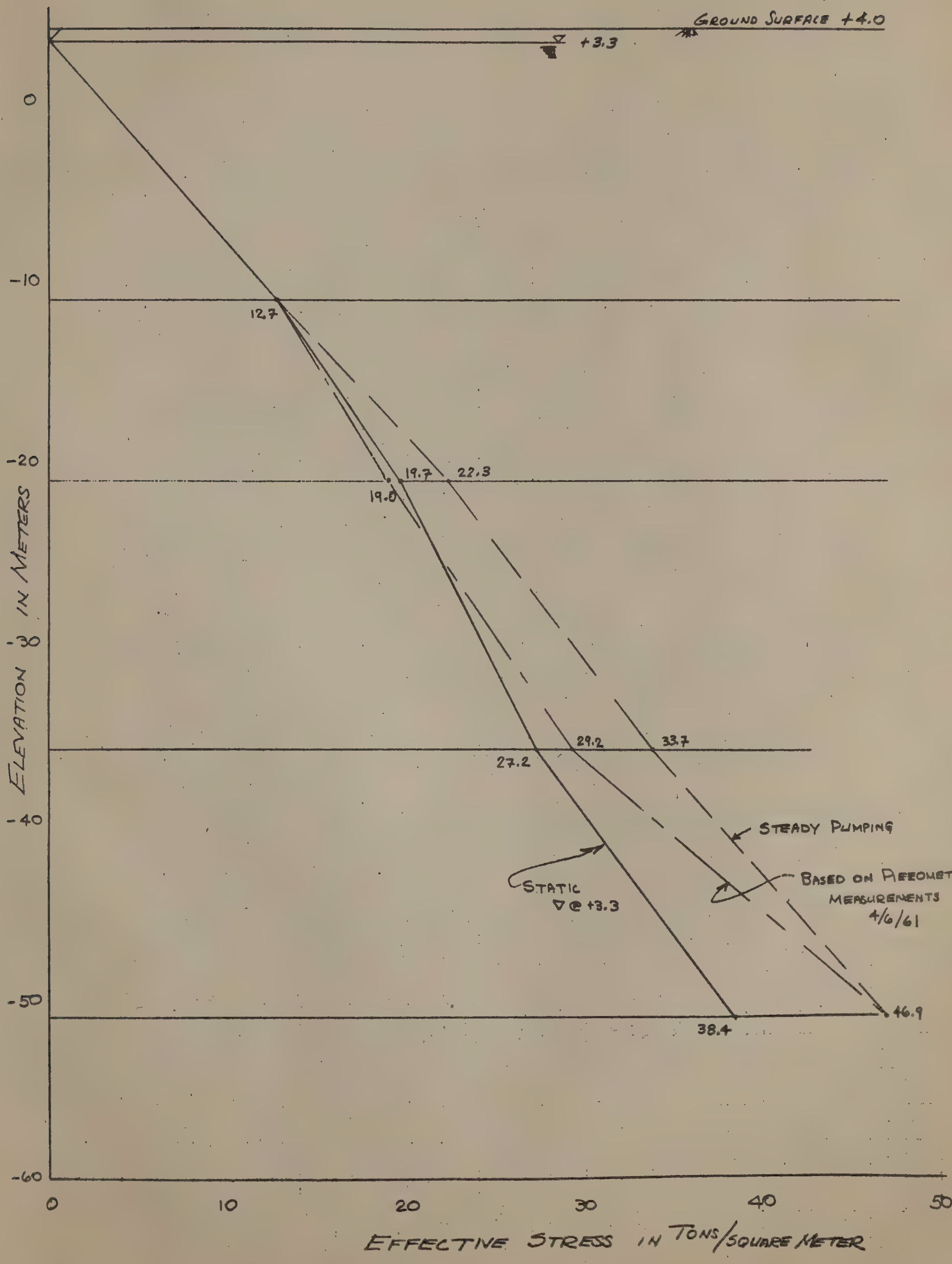


FIG. 25

After numerous conferences they decided to use a design based on $\frac{S_u}{\bar{\sigma}_{1c}} = .28$

This is a good demonstration of the fact that with all the specialized methods of testing used, the final design strength is based a good deal on judgment.

B. Oil Tank Foundation Problem-Lagunillas, Venezuela

This project has been written up as a paper for ASCE and is Part E of the Course Notes. The soil profile consists of 30 feet of silt over a 15 foot layer of clay. The area was preloaded with a 35 foot surcharge. The clay was overstressed by a factor of 4. When the last increment was placed there was 3 foot of settlement in one day and Lambe says this was due to shearing strains. They used five different methods of computing pressure distribution and picked the one that agreed with the initial excess pore pressure as the valid one.

C. Conclusions

Prof. Lambe and company realize that theories for explaining soil behaviors may be developed analytically and confirmed by laboratory testing in some cases. However, the ultimate test is in the field and that is what they are trying to accomplish on these projects. The general consensus is that it will take a long time and maybe impossible to evaluate all the properties of a soil deposit in nature relative to strength and settlement behavior of the material.

Probably one of the most practical evaluations of this problem was written by P. C. Rutledge and F. C. Walker in their conclusions of their moderators report on the Boulder Shear Strength Conference. It reads as follows:

"It seems to the writers that the present status of practical applications of the shear strength of cohesive soils and the probable developments of the next few years can be summarized in the following three points

1. In natural undisturbed cohesive soils the results of laboratory shear tests will probably never be more than a guide to engineering judgment because of the complexity of natural geologic formations and necessary limitations on numbers of samples and tests. It should be noted that every practical problem must be studied in terms of all of the enviromental variables of which the laboratory test shear strengths are only one.
2. Within specific geographic areas and geological formations, such as the pleistocene clays of the Gulf Coast and the Mississippi River Valley or the varved glacial lake deposits of the New York-Long Island Sound area, where the results of laboratory tests are backed up by the empirical evidence of structure performance in the field, results of laboratory tests can be used with confidence. The great promise in future applied soil mechanics is the collection and correlation of field evidence in terms of specific geologic formations.

3. Shear strength parameters determined by analyses of full-scale performance in the field, if applied to new design problems in the same formation by the same methods of analysis, yield reliable results, even if there are theoretical defects in the strength parameters and the analysis methods used. A major contribution of research into the physical phenomena of soil shear lies in improved understanding and reliability of analyses of field performance so that the results can be applied to new and larger projects.

It should be clear that those who came to the Conference expecting a simple answer for practical applications of shear strength data were asking too much. Those who went away from the Conference without having developed a better understanding of the complexity of the problem and the dangers of simple answers secured too little benefit."